An Interior-Point Method for Multifractional Programs with Convex Constraints

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Abstract. We present an interior-point method for a family of multifractional programs with convex constraints. The programs under consideration consist of minimizing the maximum of a finite number of linear fractions over some convex set. First, we present a simple short-step algorithm for solving such multifractional programs, and we show that, under suitable assumptions, the convergence of the short-step algorithm is weakly polynomial in a sense specified below. Then, we describe a practical implementation of the proposed method, and we report results of numerical experiments with this algorithm. These results suggest that the proposed method is a viable alternative to the standard Dinkelbach-type algorithms for solving multifractional programs.

Key Words. Multifractional programming, convex sets, interior-point methods, self-concordant barrier functions, short-step algorithms, polynomial-time convergence, predictor-corrector step.

1. Introduction

Karmarkar's 1984 landmark paper (Ref. 1) about a practical polynomial-time interior-point algorithm for linear programs triggered extensive research efforts into interior-point techniques for linear and nonlinear optimization problems; since 1984, numerous new algorithms have been proposed. As polynomial-time interior-point methods for solving linear

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programs have been generalized to solve certain classes of nonlinear programs with the same rate of convergence, but without being able to compute the exact optimal solution in polynomial time (we refer the reader to Refs. 2–8 and the references given there), the need for classifying such algorithms in a more general framework became evident; see, e.g., Refs. 7 and 9. The traditional notion of polynomiality based on the bit complexity model is no longer applicable for most nonlinear programs, and in Refs. 7 and 9 somewhat weaker notions of polynomiality (that do not require the algorithm to find the exact solution) have been proposed. Here, we will call an algorithm weakly polynomial if its rate of convergence is polynomial in the dimension of the variable \( x \) and the number of constraints.

In Ref. 10, we developed a weakly polynomial interior-point method for a class of fractional programs, namely, the minimization of a single fraction of linear functions subject to convex constraints. We stress that such programs are not convex, but only pseudoconvex. In this paper, we extend the approach of Ref. 10, and we present a weakly polynomial interior-point method for a class of multifractional programs, namely, the minimization of the maximum of \( l \) linear fractions subject to convex constraints. We remark that the multifractional case \( l > 1 \) is especially important in practice, since many applications give rise to such programs; examples include the von Neumann model of an expanding economy (Ref. 11), goal programming and multicriteria optimization (Refs. 12–14), and discrete rational Chebyshev approximation (Ref. 15).

We present some theoretical results for a short-step version of our interior-point method. In particular, we show that, under suitable assumptions, the algorithm is weakly polynomial. The theoretical results extend our earlier convergence proof (Ref. 10) for the single-fraction case \( l = 1 \) to the multifractional case \( l > 1 \). As in Ref. 10, our analysis is based on the concept of self-concordant barrier functions introduced by Nesterov and Nemirovsky (Ref. 7). In particular, the rate of convergence of our algorithm depends only on the number \( l \) and on the self-concordance parameter of the logarithmic barrier function associated with the feasible set of the multifractional program. The second key property that is used in our analysis is a Lipschitz continuity of the gradient and the Hessian of the barrier function.

We remark that interior-point methods for solving a class of fractional programming problems with convex constraints are also presented in Refs. 16 and 17. However, in Ref. 17, there is no complexity analysis. In Ref. 16, the number of iterations is shown to be polynomial; however, each iteration itself performs a finite number of dichotomy steps, and no polynomial bound on this finite number is given. The present paper differs substantially from Refs. 16 and 17 in several respects: the proposed method is different; we