TECHNICAL NOTE

Effect of Rounding Errors on the Variable Metric Method

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Abstract. It has become customary to compare the performance of unconstrained optimization algorithms on families of extended symmetric test functions. In this paper, results are presented which indicate that the performance of the variable metric algorithm on such functions is greatly distorted by rounding errors that destroy the special nature of these functions. A simple method of overcoming this difficulty is demonstrated, and it confirms the theoretical result that the number of iterations required to solve such problems is independent of the dimension.

Key Words. Unconstrained optimization, variable metric methods, quasi-Newton methods, rounding errors.

1. Introduction

In the 1960s and early 1970s, when unconstrained optimization algorithms were first being introduced and designed, a number of test problems became popular. These included the Rosenbrock function,

\[ F(x) = 100(x_2 - x_1^2)^2 + (1 - x_1)^2, \]

and the Powell function,

\[ F(x) = (x_1 - 10x_2)^2 + 5(x_3 - x_4)^2 + (x_2 - 2x_3)^4 + 10(x_1 - x_4)^4. \]

Later, when the requirement arose to test problems in higher dimensions, these simple functions were extended to families of functions, for example

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the extended Rosenbrock function,

\[ F(x) = \sum_{i=1}^{n/2} 100(x_{2i}^2 - x_{2i-1}^2) + (1 - x_{2i-1})^2, \]

and the extended Powell function,

\[ F(x) = \sum_{i=1}^{n/4} (x_{4i-3} - 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 + (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-3} - x_{4i})^4. \]

At the same time, it was quite common to extend the starting points, \((-1.2, 1.0)\) in the case of the Rosenbrock function and \((3.0, -1.0, 0.0, 1.0)\) for the Powell function, in a symmetric way; so the starting point for the extended Rosenbrock function in eight dimensions became \((-1.2, 1.0, -1.2, 1.0, -1.2, 1.0, -1.2, 1.0)\).

In 1973, Dixon (Ref. 1) commented that, under such circumstances, students were obtaining results that indicated the number of iterations required for convergence was roughly independent of dimension whether using the modified Newton, quasi-Newton, or conjugate gradient algorithms. These results have been obtained in BASIC on a DEC system 10 that used a long word length. It is indeed possible to show (Ref. 2) that, if infinite length arithmetic is used, the line search is independent of \(n\), and the terminating criterion is proportional to \(n\), then the number of iterations should be independent of \(n\). In most implementations, the line search is independent of \(n\), but the terminating criterion is not increased in proportion to \(n\), so even in infinite length arithmetic the result would only be approximate.

The current interest and results described in this note arose while implementing and testing a variable metric code in OCCAM2 for use on a transputer. The method chosen used the Broyden, Fletcher, Goldfarb, and Shanno (BFGS) update formula and a line search that combined a parabolic prediction with the Armijo method (Ref. 3).

2. Results

When running the algorithm on the extended functions from their symmetric starting points, it was noticed that, after a few iterations, the expected symmetry in both the search direction and the approximate inverse Hessian was lost in the lowest significant figures and, after typically ten to fifteen iterations, the symmetry was completely lost. This had the result that the number of iterations required for convergence increased rapidly with dimension.