The influence of snow viscosity on forced flexural-gravity waves in a basin with a density jump*

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Abstract — The effect of the internal friction of snow covering floating ice on small-amplitude waves generated by a periodic-in-time source in a basin with a density jump is considered. The dependences of amplitude-phase characteristics of the wave disturbances induced by surface and internal waves on the snow viscosity coefficient, the oscillation frequency, and the depth of the source are studied.

In this paper, the dependence of forced wave disturbances in a basin with a density jump on the internal friction of snow covering floating ice is studied.

Surface and internal waves generated by periodic disturbances in an ice-covered basin with snow absent have been examined in refs 1-3.

1. Assume that solid elastic ice covered with snow is floating on the surface of a two-layer ideal incompressible fluid having finite depth $H$. Consider the wave disturbances generated by a source situated inside the fluid at point $(0,z_0)$, whose potential is as follows:

$$\varphi^* = \frac{M}{4\pi} \ln \left\{ \frac{x^2 + (z - z_0)^2}{x^2 - (z - z_0)^2} \right\} \exp(i\sigma t),$$ (1.1)

where $M$ is the power of the source. The source comes into action at the time $t = 0$. Prior to this the fluid is undisturbed, and the ice-water interface $\zeta_1$, as well as the interface between the fluid layers, $\zeta_2$, is horizontal.

With the ice cover modelled by an elastic plate [1], we obtain equations to determine the fluid velocity potentials, $\varphi_1$ and $\varphi_2$ in the upper and lower layers, respectively:

$$\Delta \varphi_1 = 0, \quad 0 \leq z \leq H_1, \quad \Delta \varphi_2 = 0, \quad -H_2 \leq z \leq 0$$ (1.2)

with boundary conditions

$$L \varphi_1 = 0, \quad z = H_1, \quad \frac{\partial \varphi_2}{\partial z} = 0, \quad z = -H_2,$$ (1.3)

$$\frac{\partial^2}{\partial z^2} (\varphi_2 - \gamma \varphi_1) + g z \frac{\partial \varphi_2}{\partial z} = 0, \quad \frac{\partial \varphi_1}{\partial z} = \frac{\partial \varphi_2}{\partial z}, \quad z = 0$$

and with the initial ones

$$\zeta_1 = \zeta_2 = \varphi_1 = \varphi_2 = 0, \quad t = 0.$$ (1.4)

Here,

$$L = D \frac{\partial}{\partial z} + Q \frac{\partial^2}{\partial z^2} + \mu \frac{\partial}{\partial t} \frac{\partial^2}{\partial z^2} + \kappa \frac{\partial^2}{\partial t^2} \left( \frac{\partial}{\partial z} \right) + \rho_1 g \frac{\partial}{\partial z} + \rho_2 \frac{\partial^2}{\partial t^2},$$

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\[ D = \frac{E h_0^3}{12(1 - \nu^2)}, \quad \mu = \eta h^*, \quad \gamma = \rho_1 / \rho_2, \quad \varepsilon = 1 - \gamma, \]

where \( \rho_k \) and \( H_k \) are the density and the thickness of the upper \((k = 1)\) and lower \((k = 2)\) layers, respectively; \( E, \nu, \rho_0, \) and \( h_0 \) are the normal elasticity modulus, the Poisson coefficient, and the ice density and thickness, respectively; \( \eta, \rho^*, \) and \( h^* \) are the internal friction coefficient, and the snow density and thickness, respectively; and \( Q \) is the ice compression stress; \( \zeta_1 \) and \( \zeta_2 \) are related to \( \varphi_1 \) and \( \varphi_2 \) by the kinematic conditions

\[ \frac{\partial \zeta_1}{\partial t} = \frac{\partial \varphi_1}{\partial x}, \quad z = H_1, \quad \frac{\partial \zeta_2}{\partial t} = \frac{\partial \varphi_2}{\partial x}, \quad z = 0. \]  

Assume that the source is located in the upper layer. Replacing \( \varphi_1 \) in equations (1.2)-(1.5) by \( \varphi_1 + \phi^* \) and applying Fourier transform over variable \( x \) and the Laplace transformation over time, we obtain

\[ \zeta_k = M_1 \int_{-\infty}^{\infty} a_k(r) I_k \exp(i r z) \, dr, \quad k = 1, 2, \]

\[ a_1 = \cosh r H \cosh^{-1} r H_1, \quad a_2 = \gamma \cosh r (H_1 - h) \tanh r H_2 \cosh^{-1} r H_1, \]

\[ I_k = \frac{1}{2\pi i} \int_{\infty - i \infty}^{\infty + i \infty} N_k(r, \alpha) \exp(it \alpha) d\alpha, \]  

\[ N_1 = \alpha^2 \left( \gamma \tanh r H_1 \tanh r H_2 + 1 \right) + \sigma r \tanh r H_2, \]

\[ N_2 = r g (D^* + \alpha^2 x_1 - \alpha \mu_1 r^2) + \alpha^3 \tanh r (H_1 - h), \]

\[ M_1 = \frac{M}{2\pi g}, \quad D^* = D_1 r^4 - Q_1 r^2 + 1, \quad \{D_1, Q_1, x_1, \mu_1\} \sim \frac{D_1 Q_1 x_1 \mu_1}{\rho_1 g}, \]

where \( h \) is the range between the source and the layers' interface.

Bearing in mind that the fluid inhomogeneity insignificantly affects the surface wave characteristics \([2, 3]\), we will write down

\[ \Delta = \Delta_1 (r, \alpha) \cdot \Delta_2 (r, \alpha), \]

\[ \Delta_1 = \alpha^2 x^* + \alpha \mu_1 r^2 \tanh r H + D^* r g \tanh r H, \quad \Delta_2 = \alpha^2 + 2, \quad x^* = x_1 r g \tanh r H + 1, \]

\[ \gamma_2 = \sigma r g \tanh r H_1 \tanh r H_2 \left[ \tanh r (x_1 r g (\tanh r H_1 + \gamma \tanh r H_2)) \right]^{-1}, \quad H = H_1 + H_2. \]

By determining the integrals \( I_k \), we will have

\[ I_k = \sum_{n=1}^{5} N_k(r, \alpha) \tau_n^{-1} \exp(\theta_n \cdot t), \]

\[ \theta_1 = i \alpha, \quad \theta_{2,3} = \pm i \sqrt{\gamma_1} - \gamma_0, \quad \theta_{4,5} = \pm i \sqrt{\gamma_2}, \]

\[ \gamma_1 = \Delta_1 (r, \theta_1) \cdot \Delta_2 (r, \theta_1), \quad \gamma_2 = 2 \sqrt{\gamma_1 (\alpha_1,2 - i \alpha)} (\alpha_1^2 + \gamma_2), \]

\[ \gamma_4 = 2 \sqrt{\gamma_2 (\sqrt{\gamma_2} - \sigma) (\sqrt{\gamma_2} + \alpha)}, \quad \alpha_1^2 = -\gamma_0 \pm i \sqrt{\gamma_1}, \]

\[ \gamma_0 = 0.5 \mu_1 r^2 g \tanh r H x^{-1}, \quad \gamma_1 = 0.25 (4x^* D^* - \mu_1^2 r^2 g) \tanh r H r g x^{-2} \tanh r H. \]