A NUMERICAL METHOD FOR SOLVING STOCHASTIC PROGRAMMING PROBLEMS WITH MOMENT CONSTRAINTS ON A DISTRIBUTION FUNCTION

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The stochastic programming problem is considered in the case of a distribution function with partially known random parameters. A minimax approach is taken, and a numerical method is proposed for problems when information on the distribution function can be expressed in the form of finitely many moment constraints. Convergence is proved and results of numerical experiments are reported.

Keywords: Stochastic programming, incomplete information on distribution function, moment constraints, stochastic quasigradient methods.

1. Introduction

We shall consider here the following stochastic programming problem

$$\min_{x \in \mathcal{X}} \mathbb{E}_{\omega} f(x, \omega) = \min_{x \in \mathcal{X}} \int_{\Omega} f(x, \omega) \ dH(\omega),$$

where $x$ is the vector of decision variables, $x \in \mathbb{R}^n$, $\omega$ is the vector of random parameters, $\omega \in \Omega$ with $\Omega$ a Borel subset of the Euclidean space $\mathbb{R}^p$. It is assumed that the probability measure $H$ is defined on the Borel field $\mathcal{F}$ of subsets of the set $\Omega$. Problems of this type were studied in [5]. For a detailed treatment and further references, see [10,14,47]. As in [12], we use $H$ to designate both the distribution and underlying measure.

In many applications of stochastic programming there is considerable uncertainty about the distribution $H$ describing the random parameters $\omega$. In fact this is true in the majority of cases when there is no strong reason to believe that observations of $\omega$ result from the interplay of a large number of independent random factors and a resulting normal distribution. When this assumption cannot be justified or specific knowledge of the underlying physical mechanism which generates the observations is absent, the information about the distribution $H$ is usually derived from a small or medium number of observations, or from expert estimates of probabilities of future events. Often this information is not sufficient to identify the distribution $H$ uniquely.

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In the absence of full knowledge on the measure $H$ it will be assumed that a certain set $G$ is known, to which the unknown distribution $H$ of random parameters belongs, $H \in G$. The most common way to define such a set is through a finite number of moments:

$$G = \left\{ \int_{\Omega} g'(\omega)(dH_\omega) \leq 0, \ i = 1 : r, \ \int_{\Omega} g'(\omega)(dH_\omega) = 0, \ i = r + 1 : m. \right\},$$

$$H > 0, \ H(\Omega) = 1,$$

where the integrable functions $g'(\omega)$ are assumed to be known. For instance, if it is known that $a \leq \mathbb{E} \omega \leq b$ then one could take $g^1(\omega) = a - \omega, \ g^2(\omega) = \omega - b$. In the case when it is known that the probability of event $\omega \in \Omega$ is within the interval $[a, b]$ then $g^1 = a - \chi(\omega; \Omega), \ g^2(\omega) = \chi(\omega; \Omega) - b$, where $\chi(\omega; \Omega) = 1$ if $\omega \in \Omega$ and $\chi(\omega; \Omega) = 0$ otherwise.

If no other information except $H \in G$ is available it is natural to consider for a given $x$ the worst distribution in the set $G$ and consider the following problem instead of (1):

$$\min_{x \in X} \max_{H \in G} \int f(x, \omega) dH(\omega).$$

Minimax problems of this type were considered in [3,8,10,43]. Numerical techniques based on duality were developed in [12]. The inner maximization problem in (3) for a fixed $x$ is the moment problem, which was studied by many authors starting from Chebychev [26,28]. For numerical techniques, see [12,19].

Another motivation for considering (3) stems from the approximation approach for the solution of the original problem (1) and in particular for the solution of stochastic programs with recourse [5,6,22,30,34,36,41]. Even when the distribution $H$ is known it is very difficult to obtain the exact solution of (1) because objective function evaluation in (1) requires multidimensional integration. Approximation of the continuous distribution by a discrete one makes it possible to substitute summation for the integration and to develop special large scale optimization techniques. To evaluate the accuracy of such approximations, problem (3) can be used to obtain upper bounds on the solution. A similar problem yields lower bounds. By exploiting the special structure of linear programs with recourse, some techniques can obtain the solution of (3) in a finite number of steps for special types of moment functions $g'(\omega)$ [1,2,4,7,14,20,23].

Problem (3) has so far received limited attention despite its considerable applicability. The main reason is that, in general, it is even more difficult to solve than the original stochastic optimization problem (1). Except for special cases (see, e.g., [1,2,3,7,15,16,20,23,25]), the inner moment problem in (3) cannot be solved explicitly and one has to rely on numerical techniques. These techniques have to be simple enough to allow repeated usage for various values of $x$ which would be generated by a solution method for the outer minimization problem and