A SINGLE FACILITY STOCHASTIC LOCATION PROBLEM UNDER A-DISTANCE

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In this paper we consider a stochastic facility location model in which the weights of demand points are not deterministic but independent random variables, and the distance between the facility and each demand point is A-distance. Our objective is to find a solution which minimizes the total cost criterion subject to a chance constraint on cost restriction. We show a solution method which solves the problem in polynomial order computational time. Finally the case of rectilinear distance, which is used in many facility location models, is discussed.

Keywords: Chance constrained programming, facility location, A-distance.

1. Introduction

Many papers deal with stochastic facility location problems (see [1,2,5–7,11]), and some of them consider chance-constrained models, e.g., see refs. [1,6,7]. In these papers two types of models are investigated on the allowable orientations of moves. That is, in the Euclidean distance case, the allowable orientations are infinite but the shortest route is unique. In the rectilinear distance case, the allowable orientations of moves are two orthogonal ones but the shortest routes between two points exist infinitely. This case is the most popular one in the urban area model. Therefore some researchers generalize the rectilinear distance [9,10,12]. In this paper we consider a chance constrained facility location model under A-distance, which was introduced by Widmayer, Wu and Wong [12]. They use the distance for the design of VLSI. In this model, A stands for the set of allowable orientations and the cardinality of A is m (> 2). Therefore the rectilinear distance case is also included in this model. Moreover, we propose a method to solve our problem in O(n^3) computational time, where n is the number of demand points. Few other papers give a polynomial order algorithm for the stochastic facility location problem on a plane.

In the following section, we formulate our A-distance model and derive some useful properties of the problem. In section 3, we discuss the rectilinear distance model which is the most popular one in urban areas.
2. A-distance model

There are \( n \) demand points \( Q_i, i = 1, 2, \ldots, n \), on a plane. We introduce the following notations.

- \( X \): the location of the facility on the plane;
- \( d_A(X, Q_i) \): A-distance between the facility and the \( i \)th demand point;
- \( W_i \): the weight which converts the distance \( d_A(X, Q_i) \) into cost;
- \( A \): the set of \( m \) orientations \( \alpha_1, \alpha_2, \ldots, \alpha_m \).

We assume that the weights \( W_i, i = 1, 2, \ldots, n \), have independent normal distributions with positive mean \( \mu_i \) and variance \( \sigma_i^2 \). The orientations \( \alpha_i \) are assumed to satisfy

\[
0 \leq \alpha_1 < \alpha_2 < \ldots < \alpha_m < \pi.
\]

In ref. [12], an A-oriented line (or half line) is defined as the line whose orientation is one of \( \alpha_i, i = 1, \ldots, m \). The A-distance is defined as follows:

For any \( Q_1, Q_2 \in \mathbb{R}^2 \),

\[
d_A(Q_1, Q_2) = \begin{cases} 
  d_2(Q_1, Q_2) & \text{if } Q_1 \text{ and } Q_2 \text{ lie on an } A\text{-oriented line}, \\
  \min_{Q_3 \in \mathbb{R}^2} \{d_A(Q_1, Q_3) + d_A(Q_3, Q_2)\} & \text{otherwise},
\end{cases}
\]

where \( d_2(Q_1, Q_2) \) represents the Euclidean distance. Then in their paper it is shown that \( d_A(Q_1, Q_2) \) is represented by the sum of two Euclidean distances and induces a metric on \( \mathbb{R}^2 \) for any given \( A \). We modify their result to apply to the facility location model. Defining \( \alpha_{m+k} = \pi + \alpha_k, k = 1, 2, \ldots, m \), then the following inequalities hold:

\[
0 \leq \alpha_1 < \alpha_2 < \ldots < \alpha_m < \pi < \alpha_{m+1} < \ldots < \alpha_{2m} < 2\pi \leq \alpha_1 + 2\pi = \alpha_{2m+1}.
\]

Then if \( Q \) is included in the angular region which is constructed by \( X \) and the half lines \( \alpha_j \) and \( \alpha_{j+1} \), the A-distance between two points \( X \) and \( Q \) is represented as follows (see fig. 1):

\[
d_A(X, Q) = \frac{(p - x)(\sin \alpha_{j+1} - \sin \alpha_j) + (q - y)(\cos \alpha_j - \cos \alpha_{j+1})}{\sin(\alpha_{j+1} - \alpha_j)},
\]

\[
0 \leq \alpha_1 < \alpha_2 < \ldots < \alpha_m < \pi < \alpha_{m+1} < \ldots < \alpha_{2m} < 2\pi \leq \alpha_1 + 2\pi = \alpha_{2m+1}.
\]

Fig. 1. The A-orientations and the polygonal line segment realizing the A-distance between \( X \) and \( Q \).