Exchanging heterogeneous goods via sealed bid auctions and transportation systems

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A primary commodity such as wheat, rice, coffee, oil, etc., is shipped from \(m\) locations where it was grown or pumped to \(n\) manufacturers. Each manufacturer processes, packages, advertises, and distributes the commodity under a consumer product brand name. The resulting heterogeneous good is sold at a sealed bid auction, in competition with the other manufacturers of the consumer product, to \(k\) final customers. The problem to be considered in this paper is to find a way of determining prices for the goods produced and the physical exchanges between seller and buyer which satisfy flow conditions and which take into account the evaluations of the goods by both sellers and buyers. The first model for doing this is given in section 1, which combines the idea of a sealed bid auction due to Shapley, Shubik and Thompson, with a conventional transportation system. The sealed bid auction is used to determine the exchange prices, and the transportation system is used to calculate the production and transportation costs. It is suggested that the resulting model type can also be applied in a wide range of problems that arise in the marketing of goods sold under brand names (i.e., heterogeneous goods) regardless of whether they are actually exchanged at formal auctions. We show in section 6 that our model is a generalization of the transshipment model in a recent paper by Dubey and Shapley [1]. In their model they considered a number of oligopolists engaged in transshipping and trading goods. Their oligopolists set their prices in order to maximize profits, rather than having them determined by an auction process as is done in our model. In section 7, we extend the model to one in which the wholesalers are permitted to make positive profits. We show how to calculate the values of coalitions of the various players in the model.

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1. Introduction

In spite of the fact that most goods and services in a modern economy are heterogeneous, that is, possess features which have characteristics associated with the manufacturer (such as brand names, trademarks, etc.), and which compete in individual "niches" of the market, economic theory has had difficulties in finding an attractive theoretical format for capturing such realities. Too often, economists simply assume that such goods are traded in markets of perfect competition, even though the assumption that each market participant is a price taker manifestly does not apply. A general analytic format encompassing most of the modern forms of marketing of heterogeneous goods, which is mathematically comprehensive, simple and straightforward, has not heretofore been available.

The mathematical format of a sealed bid auction is suggested here as the basis for a new market paradigm which may provide what is desired. It is well known that the sealed bid auction is frequently used for the pricing of many commodities, such as furs, real estate, and oil leases. But the important observation is that the pricing of heterogeneous goods in the modern economy can be perceived as occurring as if it resulted from a sealed bid auction. That is, the sealed bid auction model will be proposed here as a mathematical model to determine the pricing of heterogeneous goods quite generally, whether or not they are actually sold under conditions of a formal sealed bid auction.

In a sealed bid auction, the sellers arrive at the market with the goods they wish to sell, each having their brand names and advertising images. Each potential buyer inspects these goods and enters sealed bids for the goods of the sellers. Naturally, each buyer will attach high bids to the goods whose design, packaging, and product image he likes, and low bids to other goods that seem to him to have less quality, less appeal or shoddy appearance. The bid prices thus indirectly reflect the buyers' relative estimation of the various heterogeneous goods offered for sale in the market. It is also assumed that the buyer announces the maximum quantity of each good that he is willing to buy, that is, his demand.

Each seller is permitted to state his reservation price, which is the lowest price at which he is willing to sell his good. Since his goods are on display, it is also clear what the maximum amount of each good is that he is offering for sale, that is, his supply. The auctioneer (which in our case will be a mathematical program) then determines the market price of the good of each seller and the actual exchanges of goods between sellers and buyers by solving the linear programming problem of the sealed bid model, as explained in section 2.

The market price paid by a given buyer cannot exceed his bid price; in the case it is less than his bid price, the difference between the bid and actual price is his "buyer's surplus". Buyers whose bids are low may receive none or only part of their demand. But in no case will a buyer be required to buy more than his stated demand. Similarly, the market price received by a given seller cannot be less than his