STRESS–STRAIN STATE OF SHELLS WITH RECTANGULAR HOLES

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The stress–strain state of shallow shells with large rectangular holes was investigated. The use of the apparatus of special discontinuous functions reduces the problem to the solution of a fourth-order differential equation for a complex deflection–force function with variable coefficients in the form of delta functions, their derivatives, and column functions. The solution is presented in the form of systems of basis functions composed of series of special functions reflecting the discontinuous character of the forces and moments and having rapid convergence both in the local zone near the hole and in continuous zones. This made it possible to construct effective calculation algorithms and to take into account stress concentration near the corners of the hole.

The general approaches to investigating shells with large rectangular holes are given in monographs [1, 2]. The use of most existing calculation methods for solving such problems is complicated by difficulties of realizing them with consideration of stress concentration near the holes.

In the present work a method is given for calculating shells with large rectangular holes based on using special discontinuous functions. The calculation algorithm constructed takes into account the irregular character of the distribution of components of the stress–strain state, and the series composing the desired solution have almost the same rapid convergence both near the hole and in the continuous region. This makes it possible to construct simple and reliable computational processes and to increase the reliability of the results.

When examining the equations describing the stress–strain state of shells with holes we will assume that the middle surface of the shells is referred to a rectangular coordinate system \((x_1, x_2, z)\). The curvatures of the shell retain a constant value. The boundaries of the hole are parallel to the contour (Fig. 1). The presence of the hole can be taken into account by the following assignment of the thickness:

\[
h^* = h - H(x_1 - a_{1h}) - H(x_2 - a_{2h});
\]

where

\[
H(x_1 - a_{1h}) = \begin{cases} 0, & x_1 < a_{1h} \\ 1, & x_1 \geq a_{1h} \end{cases}
\]

\[
H(x_2 - a_{2h}) = \begin{cases} 0, & x_2 < a_{2h} \\ 1, & x_2 \geq a_{2h} \end{cases}
\]

\[H(x_1 - a_{1h})\] is a Heaviside unit function.

Substituting (1) as the limits of integration into the relations between forces and deformations, we find the vector of internal forces:

\[
\{N^*\} = \{N\} - \{N\} H_{x_1} H_{x_2},
\]

where \(\{N\}\) is the vector of internal forces in a smooth shell.

After substituting expression (2) into the equations of equilibrium [3] and introducing the force function according to the conditions

\[
N_{x_1} = \frac{\partial^2 F}{\partial x_1^2} - \frac{\partial^2 F}{\partial x_2^2} H_{x_1} H_{x_2} \quad (1 \rightarrow 2);
\]

\[
N_{x_2} = - \frac{\partial^2 F}{\partial x_1 \partial x_2} + \frac{\partial^2 F}{\partial x_1 \partial x_2} H_{x_1} H_{x_2}
\]
we obtain the differential equation for the function of normal displacements \( w \) and \( F \):

\[
(D \Delta^2 w - \Delta_k F)(1 - H_{ii} H_{jj}) = q + \sum_{i=1}^{2} \sum_{j=1}^{2} D(L_{ij} \delta_{ii} H_{jj} - L_{ij}^1 \delta_{ii} H_{jj} + L_{ij}^2 \delta_{ii} \delta_{jj}) \quad (i \neq j),
\]

where

\[
\Delta^2 = \left( \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial x_j^2} \right)^2; \quad \Delta_k = \kappa_1 \frac{\partial^2}{\partial x_i^2} + \kappa_2 \frac{\partial^2}{\partial x_j^2};
\]

\[
\xi_i = 2 \left( \frac{\partial^2}{\partial x_i^2} + \frac{\partial^2}{\partial x_i \partial x_j} \right); \quad L_{ij}^1 = \left( \frac{\partial^2}{\partial x_i^2} + \mu \frac{\partial^2}{\partial x_j^2} \right); \quad L_{ij}^2 = (1 - \mu) \frac{\partial^2}{\partial x_i \partial x_j};
\]

\( D \) is cylindrical rigidity; \( \mu \) is the Poisson ratio;

\( \delta_{ii} = \delta(x_i - a_i) \) is the Dirac delta function.

Since the conditions of continuity of deformations are fulfilled only within the limits of the continuous part, the vector of deformations of the middle surface can be written in the form

\[
\{ \xi \} = \{ \xi \} - \{ \xi \} H_{ii} H_{jj},
\]

where \( \{ \xi \} \) is the vector of deformations of the middle surface of a smooth shell.

Having substituted (5) into the continuity relationships [3], we have

\[
\frac{1}{E_h} \Delta^2 F + \Delta_k F)(1 - H_{ii} H_{jj}) = \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{1}{E_h} (L_{ij}^2 \delta_{ii} H_{jj} -
\]

\[
-L_{ij}^1 \delta_{ii} H_{jj} + L_{ij}^2 \delta_{ii} \delta_{jj}) \quad (i \neq j),
\]

where

\[
L_{ij}^1 = \left( \frac{\partial^2}{\partial x_i^2} - \mu \frac{\partial^2}{\partial x_j^2} \right); \quad L_{ij}^2 = (1 + \mu) \frac{\partial^2}{\partial x_i \partial x_j};
\]

Carrying out complex transformation of (4) and (6), we obtain one equation for the function \( \varphi = w + \frac{i n}{E_h} F \)

\( (n = \sqrt{12(1 - \mu^2)}/h) \):

\[
L \varphi = \frac{A}{D} + L \varphi H_{ii} H_{jj} + \sum_{i=1}^{2} \sum_{j=1}^{2} (L_{ij}^1 \varphi \delta_{ii} H_{jj} - L_{ij}^2 \varphi \delta_{ii} \delta_{jj} + L_{ij}^2 \varphi \delta_{ii} \delta_{jj}) \quad (i \neq j).
\]