It is suggested to use the finite-element approach in combination with the subregion isolation procedure for modeling the spectrum of natural frequencies and dynamic stress-strain state of machine parts with damages. The effect of service damages on the natural frequencies of a real operating axial-flow compressor blade was evaluated and the theoretical stress-concentration factor was calculated.

A entire group of service damages of machine parts is related to a change in flexural rigidity as a consequence of mechanical damages, erosion, or overheating. In the general case the damages have a complex character, encompass a local portion of the material of machine parts, and depend on the causes of their occurrence. All this creates certain difficulties when describing a model of a part with damages. Attempts to investigate the regularities of the dynamics and strength of parts with damages on the basis of bar models were made earlier [1], which made it possible to analyze the effect of the site of damage on a change in the spectrum of natural frequencies of flexural vibrations. For such an approach it is expedient to use the method of initial parameters, when the transition matrix of the bar section is constructed by means of Krylov functions:

\[
\begin{bmatrix}
Y \\
y' \\
y'' \\
y'''
\end{bmatrix}_{i+1} = 
\begin{bmatrix}
S & \frac{1}{\beta^2} & \frac{1}{\beta} & \frac{1}{\beta^3} \\
\beta V & S & \frac{1}{\beta^2} & \frac{1}{\beta^3} \\
\beta^2 U & \beta V & S & \frac{1}{\beta^2} \\
\beta^3 T & \beta^2 U & \beta V & S
\end{bmatrix}
\begin{bmatrix}
Y \\
y' \\
y'' \\
y'''
\end{bmatrix}_i
\]

or in a matrix form

\[
\{ Y \}_{i+1} = [A]^{-1} \{ Y \}_i.
\]

where \( S, T, V, U \) are Krylov functions,

\[
\begin{align*}
S(\beta x) &= \frac{1}{2}(\cos \beta x + \sin \beta x); \\
T(\beta x) &= \frac{1}{2}(\sin \beta x + \cos \beta x); \\
U(\beta x) &= \frac{1}{2}(-\cos \beta x + \sin \beta x); \\
V(\beta x) &= \frac{1}{2}(-\sin \beta x + \cos \beta x);
\end{align*}
\]

\( \beta^4 \) is a frequency parameter

\[
\beta^4 = \frac{\rho^2 A}{E J}.
\]

Service damage was modeled by reducing the width of the bar on one of the sections. An examination of such a model makes it possible primarily to analyze the effect of the site of damage on a change in the spectrum of natural frequencies of flexural vibrations, particularly at lower frequencies, which in vibroacoustic inspection are the most informative. Along with this, on such a simple model it is possible to analyze the most common regularities of the distribution of dynamic stresses in the case of the presence of damages.
Cantilever bars measuring 0.07 x 0.0175 x 0.004 m were used as the models. Damage was modeled by removing a part of the material symmetrically relative to the axis of the bar. The results of calculating the natural frequencies of such models are given in Table 1 for four lower modes of flexural vibrations. Two opposite factors have a substantial effect on the formation of the spectrum of natural frequencies in the presence of damage. On the one hand, service damage, modeled by removing material, leads to a decrease of flexural rigidity and causes a decrease of natural frequencies. On the other hand, as a result of removing a part of the material the effective mass of the cross section decreases and the natural frequency increases. The effect of the first factor is most substantial for damages located in the stressed zone, in particular, near the restraint for the first mode of vibrations, and the effect of the second for damages located in the unstressed zone with considerable amplitudes of deformations, for example, the peripheral part of the cantilever bar for the first mode of flexural vibrations. The total picture of the mutual effect of these two factors depends on the mode of vibrations and distribution of dynamic stresses. If the damage is located in the zone of maximum stresses, then the frequencies decrease. Figure 1 shows the effect of the site of damages on the relative dynamic stresses for the first and second modes of flexural vibrations.

The most complete and reliable results of modeling natural frequencies of damaged machine parts can be obtained with the use of a specialized finite-element program system [2]. The presence of damages causing a decreases of rigidity of the investigated structure can be represented by a change in the corresponding parameters or by breaking of the bonds between adjacent elements as a result of weakening of the structure, by introducing additional points when modeling a crack, and by changing the thickness at the finite-element mesh points.

The finite-element approach makes it possible to solve the following interrelated problems for machine parts with service damages:

- numerical determination of significant changes in natural frequencies as a result of damage and constructing a diagnostic matrix for subsequent recognition of damages from experimental data [3];
- analysis of stress concentration in the zone of damage to determine the maximum allowable damages;
- analysis and optimization of the distribution of dynamic stresses over elements of the machine to reduce them in the zone of possible service damages and to increase the natural frequency.

The subsystem of calculating the dynamics and strength of plates and shells on the basis of a plane bending-membrane triangular finite element, which permits analyzing curved shells of variable thickness, has obtained the greatest practical use. The L-coordinates can be used when approximating membrane movements [2]:

\[ N_1 = L_1; \quad N_2 = L_2; \quad N_3 = L_3, \quad (5) \]

where

\[
\begin{align*}
L_1 &= \frac{1}{2\Delta}(a_2 + b_2x + c_2y); \\
L_2 &= \frac{1}{2\Delta}(a_1 + b_1x + c_1y); \\
L_3 &= \frac{1}{2\Delta}(a_3 + b_3x + c_3y); \\
L_1 + L_2 + L_3 &= 1;
\end{align*}
\]

\[(6, 7)\]

### Table 1. Natural Frequencies of Vibration (Hz) of a Bar with Damage

<table>
<thead>
<tr>
<th>l, mm</th>
<th>1u</th>
<th>(\Delta f,%)</th>
<th>2u</th>
<th>(\Delta f,%)</th>
<th>3u</th>
<th>(\Delta f,%)</th>
<th>4u</th>
<th>(\Delta f,%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>651.0</td>
<td>-6.15</td>
<td>4159</td>
<td>-4.32</td>
<td>1790</td>
<td>-3.15</td>
<td>23310</td>
<td>-2.28</td>
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<tr>
<td>10</td>
<td>666.0</td>
<td>-3.99</td>
<td>4353</td>
<td>0.07</td>
<td>12280</td>
<td>0.88</td>
<td>24012</td>
<td>0.56</td>
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<tr>
<td>20</td>
<td>678.8</td>
<td>-2.18</td>
<td>4374</td>
<td>0.62</td>
<td>12183</td>
<td>0.08</td>
<td>23851</td>
<td>0.0</td>
</tr>
<tr>
<td>30</td>
<td>689.2</td>
<td>-0.65</td>
<td>4322</td>
<td>-0.58</td>
<td>12175</td>
<td>0.0</td>
<td>23734</td>
<td>-0.42</td>
</tr>
<tr>
<td>40</td>
<td>698.6</td>
<td>0.71</td>
<td>4284</td>
<td>-1.45</td>
<td>12085</td>
<td>0.87</td>
<td>23334</td>
<td>-0.08</td>
</tr>
<tr>
<td>50</td>
<td>708.4</td>
<td>2.12</td>
<td>4297</td>
<td>-1.15</td>
<td>11965</td>
<td>-1.71</td>
<td>22539</td>
<td>-0.90</td>
</tr>
<tr>
<td>60</td>
<td>719.8</td>
<td>3.76</td>
<td>4050</td>
<td>1.24</td>
<td>12138</td>
<td>-0.29</td>
<td>22380</td>
<td>-1.15</td>
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<tr>
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<td>693.7</td>
<td>0.0</td>
<td>3980</td>
<td>0.0</td>
<td>12173</td>
<td>0.0</td>
<td>23854</td>
<td>0.0</td>
</tr>
</tbody>
</table>

85