DETERMINATION OF CYCLE NONPROPORTIONALITY COEFFICIENT

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We propose a cycle nonproportionality coefficient for a broad class of complex cyclic paths, allowing us to establish a clear relationship between the geometry of the strain path and the maximum hardening level attainable in the material in the steady state for a fixed value of the maximum plastic or total strain range. We discuss the effectiveness of using the proposed cycle nonproportionality coefficient compared with analogous parameters familiar from the literature. Based on a previously developed version of the endochronic theory of plasticity, we have numerically modeled the behavior of a material for cyclic loading along different planar nonproportional paths.

Cyclic loadings encountered in technology in most cases are multiaxial and vary in a nonproportional manner. Accordingly, work devoted to investigation of the behavior of materials under complex cyclic loading conditions has become very important today. Appreciable progress has been made in the area of nonproportional cyclic plasticity due to a number of experimental investigations determining the important effects which arise under complex cyclic loading and often are not observed under simple loading of variable sign [1-4]. Many papers have been devoted to the study of the effect of the loading history, the amplitude and shape of curvilinear or piecewise-linear cyclic strain paths on the deformation properties and the useful life of construction materials. The major results of these investigations are summarized as follows.

Under complex cyclic deformation, the material is hardened much more than for proportional deformation; in this case, the degree of hardening increases with an increase in the complexity of the path (for the same maximum deformation amplitude) and may be 1.5-3 times greater than for simpler deformation. At the same time, a significantly smaller change in the stress state often leads to a change in the useful life by an order of magnitude or more. Such appreciable strain hardening does not correlate with simple measures of the deformation such as the length of the arc of the strain path in a cycle, the length of the arc of plastic strain, the work of plastic deformation, etc. [5]. This circumstance has stimulated work studying the effect of the cycle geometry on the strain hardening. Two different approaches have been formulated for solution of this problem. According to the first approach, the equations of state in the theory of plasticity should take into account the complexity of the loading process: the curvature of the path, the points of inflection and discontinuity, their effect on the deformation process [6]. The second approach proposes use of a nonproportionality coefficient, an integral characteristic of the cycle geometry [7, 8].

One of the first attempts to characterize the shape of the cycle and to take into account the effect of the direction of loading in order to account for the additional hardening arising upon deformation along a circular path should be considered introduction of the rotation coefficient in [2]. Further development of investigations in this direction is connected with work by McDowell, who introduced the concept of the cycle nonproportionality effect and proposed a continuous (instantaneous) measure of the nonproportionality \( \lambda \) as applied to the strain rate tensor [9] and subsequently to the plastic strain tensor [7]:

\[
\lambda = \frac{\int_{t_1}^{t_f} \left( \mathbf{N} \cdot \frac{d\mathbf{e}}{dt} \right) dt}{\int_{t_1}^{t_f} \left| \frac{d\mathbf{e}}{dt} \right| dt},
\]

TABLE 1. Values of the Nonproportionality Coefficients and Maximum Equivalent Stresses for Different Cyclic Plastic Strain Paths

<table>
<thead>
<tr>
<th>No. of cycle shape</th>
<th>Shape of cycle</th>
<th>$\varepsilon_{\text{max}}$ %</th>
<th>$\Phi$ [7]</th>
<th>$\Phi$ [7]</th>
<th>$\sigma_{m}^{\text{calc}}$, MPa [11]</th>
<th>$\sigma_{m}^{\text{exp}}$, MPa</th>
</tr>
</thead>
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<tr>
<td>1 *</td>
<td></td>
<td>0.2</td>
<td>0</td>
<td>0</td>
<td>290</td>
<td>290</td>
</tr>
<tr>
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<td>0.624</td>
<td>0.6366</td>
<td>417</td>
<td>410</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>0.2</td>
<td>0.926</td>
<td>0.810</td>
<td>458</td>
<td>440</td>
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<tr>
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<td>0.850</td>
<td>0.890</td>
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<td>465</td>
</tr>
<tr>
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<td>1</td>
<td>1</td>
<td>490</td>
<td>490</td>
</tr>
</tbody>
</table>

*Here and in Table 2, the data from the baseline experiments are marked with an asterisk (*).

Fig. 1. Cyclic strain paths.

where $N$ is the unit vector in the direction of the maximum plastic strain range; $\xi_i$ and $\xi_f$ are the accumulated plastic strain at the beginning and at the end of the loading block. According to [7], the nonproportionality coefficient is defined by the expression

$$\Phi = \left| 1 - \frac{\pi}{2} \frac{\lambda - \frac{2}{\pi}}{\frac{2}{\pi}} \right|.$$  

(2)

Despite the fact that a nonproportionality coefficient in the form of (2) has been used successfully in [7, 10], some negative aspects of this parameter have been noted in [11]. Simultaneously the authors of [8], in analysis of the experimental results, proposed defining the degree of nonproportionality of the cycle as a quantity equal to the angle between the vectors for the rate of variation in the stresses and the rate of variation in the plastic strain:

$$A = 1 - \cos^2 \theta,$$  

(3)

where

$$\cos \theta = \frac{\langle \dot{\varepsilon} \cdot s \rangle}{\left( \left| \dot{\varepsilon} \right| \cdot \left| s \right| \right)}$$  

(4)

is the projection of the component of the normalized vector $\dot{\varepsilon}$ in the direction perpendicular to the normalized vector $s$. Then this parameter was used in [12]. However, the idea remained attractive that the accuracy of the calculated values of the harden-