STEADY-STATE MODE-LOCKING OPERATION IN A
LASER WITH A SATURABLE ABSORBER

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It is shown that a simple correspondence exists between the form of radiation for a steady-state mode-locking operation and the well-known solution of the one-mode problem. The dependence of the amplitude and shape of the pulses on the amount of absorber is determined.

It is well known that stationary lasing in a laser may become unstable when a saturable absorber is introduced into its cavity resonator. In [1, 2] it was shown that instability at the frequencies of intermode beats usually develops for smaller amounts of the absorber than is required for instability of slow modulations (i.e., modulations having a characteristic time much longer than the time required to traverse the cavity). It is natural to expect that the development of instability at the frequencies of the intermode beats will lead to the establishment of a periodic operating mode with a characteristic modulation time shorter than or of the order of $L/c$ ($L$ is the cavity length; $c$ is the velocity of light); this mode may be interpreted as mode locking.*

The purpose of the present paper is to perform a theoretical determination of the principal characteristics of such a periodic operating mode.

Let us consider a one-dimensional traveling-wave cavity resonator (Fig. 1); an active substance 1 and a saturable absorber 2 which we shall consider to be inertialess are placed in it.

Usually the signal variation during one pass in lasers operating in a quasi-continuous-wave mode is small. We shall seek the solution in the form of a quasisinusoidal traveling wave having a frequency equal to the frequency of the working transition of the active substance and with a phase that is constant in time. Under these conditions the partial differential equations for the field amplitudes $E_0$, the nondiagonal element of the density matrix $\sigma$ and the population difference of the working levels $n$ of the active substance may be transformed to a system of differential-difference equations incorporating ordinary derivatives if the boundary conditions are taken into account:

$$E_0 \left( t + \frac{L}{c} \right) - E_0(t) = -(1 - r) E_0(t) + \frac{2\pi r \omega \mu_1 \Delta_1}{c} \phi(t) + \frac{\pi \hbar \omega r \Delta_2}{2cT_{12}} \frac{n_{12} E_0(t)}{1 + \beta E_0(t)},$$

$$\frac{dn}{dt} + \frac{n - n_{12}}{T_{11}} = -\frac{4\mu_1}{\hbar} E_0,$$

$$\frac{d\sigma}{dt} + \frac{\sigma}{T_{21}} = \frac{\mu_2}{\hbar} n E_0.$$  

Here $\hbar$ is Planck's constant; $r$ is the reflection coefficient of the mirrors; $\omega$ is the light frequency; $\beta = 4\mu_2^2 T_{12} T_{22}/\hbar^2$; $T_{11}$ and $T_{12}$ are the relaxation times of the populations; $T_{21}$ and $T_{22}$ are the relaxation times of the nondiagonal elements of the density matrix; $n_{12}$ and $n_{22}$ are the densities of the total number of working molecules; $\mu_1$ and $\mu_2$ are the dipole moments; $\Delta_1$ and $\Delta_2$ are the lengths of the samples of active substance and the saturable absorber, respectively. The last term in the right side of the first equation of this system describes the effect of the nonlinear absorber.

* In [3], for example, a saturable absorber was introduced into the cavity of a laser operating in a continuous-wave mode. Under these conditions periodic pulses whose length was much shorter than $L/c$ were observed.

An analysis of the system (1) in general form presents significant difficulties. However, in those cases when the length of the radiation pulse is $T_{\text{pul}} \gg T_{21}$, this system may be simplified. Namely, by expressing $\sigma$ from the third equation in terms of $E_0$ with an accuracy of up to terms of order $T_{21}/T_{\text{pul}}$ inclusive* and substituting this expression into the first equation, we obtain

$$E \left( t + \frac{\Delta}{c} \right) E(t) = \left[ \frac{B_1 \Delta_1}{c} n(t) - \frac{B_2 \Delta_2}{c} \frac{n_{02}}{1 + B_2 T_{12} E_0^2(t)/\hbar \omega} - (1 - r) \right] E(t) - \tau \left( \frac{dE(t)}{dt} - T_{21} \frac{d^2E(t)}{dt^2} \right).$$

Here

$$E^* = \frac{2}{\pi} E_0^2, \quad \tau = \frac{B_1 \Delta_1 \bar{n} T_{11}}{c}, \quad B_{1,2} = \frac{2\pi r \omega k_2 T_{21,22}}{\hbar \omega}.$$

are the stationary values of the populations and field intensity.

Although the target of our investigation is the "fast" (in comparison with $L/c$) solutions of the system (2), we shall begin by considering the "slow" solutions (i.e., the solutions that vary slowly in a time $L/c$). Then $E(t + \Delta/c) \approx E(t) + (L/c)dE(t)/dt$. Taking account of the fact that $L/c \gg \tau, T_{21}$, Eqs. (2) may be written in the following form for this case:

$$\frac{dm}{dt} = 2 \left[ \frac{B_1 \Delta_1}{c} n - \frac{B_2 \Delta_2}{c} \frac{n_{02}}{1 + B_2 T_{12} E/m} - (1 - r) \right] m,$$

$$\frac{dn}{dt} + \frac{n - n_{01}}{T_{11}} = -B_1 \bar{n} m,$$

where $m = E^2/\hbar \omega$ is the density of the number of photons.

Equations (3) coincide with the balance equations which describe a one-mode laser with a saturating absorber. In [4] such equations were investigated for $n_{02} = 0$ (i.e., in the absence of an absorber). There it was shown that the solutions may be found in the quasiconservative approximation — i.e., the solutions of such a system are close to the solutions of the system of equations

$$\frac{dm}{dt} = \frac{\alpha}{L/c} (n - \bar{n}) m,$$

$$\frac{dn}{dt} = B_1 \bar{n} (m - m),$$

where $\alpha = 2B_1 \Delta_1/c$, $\bar{m} = E^2/\hbar \omega$.

This system has periodic solutions. The transients in the system (3) may be treated as a slow decay of the solutions of the system (4) for $n_{02} = 0$.

Since the instability of stationary lasing is achieved for fairly small quantities of the absorber, it may be expected that for $n_{02} \neq 0$ the solutions of (3) will be close to the solutions of (4) for $n_{02} \neq 0$ also.

* Under these conditions the terms with a multiplier of order $(n - \bar{n})/\bar{n}$ are dropped. As will be shown below, this quantity is much less than unity.