\( \gamma \) (see Sec. 1.10) to the vectors \( h^\alpha = h^\alpha \oplus 0 \) (where \( h^\alpha = Dx \)):

\[
D(x, x) = \langle F\omega x, F\omega x \rangle_{H^\alpha} + \langle I\omega x, I\omega x \rangle_{H^\omega} + \langle P_{(H^\omega), Dx, Dx} \rangle, \tag{2.24}
\]

here \( a^\omega (\zeta) \) is the orthoprojection onto the subspace of the isolated part of the colligation \( \alpha \); \( \omega \) is the measure corresponding to the function \( \varphi, \psi \), defined by the formula (1.28); by virtue of formula (1.29), \( I^\omega x \) has the form [by the same symbol we have denoted in (2.24) also the corresponding vector measure]:

\[
(I^\omega x)(\zeta) = \left[ \begin{array}{cc} \omega \varphi & 0 \\ 0 & \omega \psi \end{array} \right] \cdot G^\alpha (\zeta) Dx - \int \frac{l_1 - |\zeta|^2}{\gamma} \left[ \begin{array}{cc} \psi^* & \omega \varphi \\ \omega^* \varphi & \psi \end{array} \right] \left[ \begin{array}{cc} 1_{L}\n \omega \n 1_{L} \end{array} \right] F^\omega x dm, \tag{2.25}
\]

where \( G^\alpha \) is the Fourier transform connected with the simple part of the colligation \( \alpha \); \( \varphi, \psi \) are determined in terms of \( \omega \) by the formulas (1.22) and (1.22'), \( \varphi, \psi \) by the formulas (1.26), and the inner product in \( H^\omega \) is defined by the Hellinger integral.

From formulas (2.9) there follows that

\[
G^\alpha DTx = tG^\alpha Dx - \left[ \begin{array}{c} 1_{N}\n S \\
S^* \n 1_{N}^* \end{array} \right] \left[ \begin{array}{c} 0 \\
-\frac{Mx}{\omega} \n -\frac{\omega^*}{E x} \end{array} \right]. \tag{2.26}
\]

From here, in particular, there follows that \( I\omega x = tI\omega x \).

Property (2.26) allows us to transform the expression for \( I^\omega x \) in some important special cases: let \( \zeta (|\zeta| > 1) \) be such that there exist \((\zeta - T)^{-1}\) and \((1_x - \zeta T)^{-1}\); then

\[
(I^{\omega x})(\zeta) = \left[ \begin{array}{cc} \psi^* & \omega \varphi \\ \omega^* \varphi & \psi \end{array} \right] \cdot \left[ \begin{array}{c} E(1_x - \zeta T)^{-1} \n x \\
-\frac{M(1_x - \zeta T)^{-1} x}{\zeta} \n -\frac{\zeta - \zeta^2}{\zeta} \n \zeta \n \zeta \n \zeta \end{array} \right] \left[ \begin{array}{cc} \psi^* & \omega \varphi \\ \omega^* \varphi & \psi \end{array} \right] \left[ \begin{array}{c} 1_{L}\n \omega \n 1_{L} \end{array} \right] F^\omega x dm. \]

As shown in Sec. 1.9, we have \( I^{\omega x} = 0 \) (\( \forall x \in X \)) if and only if \( \omega = 0 \) [i.e., \( a^\omega (\zeta) = 0 \)].

The last term in (2.24), corresponding to the isolated part of the colligation \( \alpha \), occurs in an unremovable manner and does not depend on the selection of the parameter \( \omega \).

### STABILITY IN THE I. V. OSTROVSKII—R. CUPPENS THEOREM ON GROUPS

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Let \( X \) be a locally compact separable metric Abelian group. Unimprovable estimates are obtained for the stability of the decompositions of the generalized Poisson distribution \( e(F) \) on \( X \), where \( F \) is a completely finite measure on \( X \) such that its powers \( F^m \) with respect to convolution are pairwise singular for distinct natural numbers \( n \).

I. V. Ostrovskii and R. Cuppens [1, p. 258 of the Russian edition] have proved the following theorem: if an \( n \)-dimensional infinitely divisible (i.d.) distribution \( F \) does not have Gaussian components and its Levy spectral measure \( F \) is completely finite and concentrated on a set with independent points, then \( P_{(N)} \). Under various additional restrictions on the measure \( F \), this fact has been proved by D. A. Raikov, I. V. Ostrovskii, R. Cuppens, G. P. Chistyakov. Although the author has proved in [2] the above-mentioned theorem under the restriction \( F(R^m) < \ln 2 \), the method of investigation, developing some ideas of Ramachandran, has turned out to be very general and has allowed Fel'dman [3] to carry over the I. V. Ostrovskii—R. Cuppens theorem to a locally compact, separable, Abelian metric group in the following formulation.

Let \( F \) be a completely finite measure on a group \( X \) such that its powers with respect to convolution are pairwise singular \( F^m \) for any natural numbers \( n \neq m \). Then the generalized
Poisson distribution $e(F) = \exp\{-F(X)\} \left( E_0 + F + \frac{1}{2!} F^2 + \cdots \right)_+, \text{ where } E_0 \text{ is the distribution concentrated at the point } 0, \text{ belongs to the class } I_0.

In this note we give stability estimates for this theorem; for this we had to improve and simplify the method of investigation in the author's paper [2]. We describe the stability effect of the decompositions of the distribution $\mu$ on the group $X$ in terms of the Zolotarev characteristic $\beta_{d, a}(\mu, \mu, \epsilon) > 0$ [4]. Let $d_1$, $d_2$ be metrics in the space of the distributions $\mu$ on the group $X$, let $K_0$ be the set of the components of the distribution $\mu$, and let $B(\epsilon, \mu) = \{ \nu \text{ is a distribution: } d_1(\nu, \mu) < \epsilon \}$. We denote 

$$\beta_{d, a}(\epsilon, \mu) = \sup_{\nu \in B(\epsilon, \mu)} \sup_{\mu \in K_0} \inf_{\nu} d_2(\mu, \nu).$$

We shall say that for the distribution $\mu$ one has the stability effect of the decompositions in the metrics $(d_1, d_2)$ if $\beta_{d, a}(\epsilon, \mu) \rightarrow 0, \epsilon \rightarrow 0$. Ushakov [5] has investigated the stability effect of distribution decompositions on separable complete metric groups without stability estimates.

For the metric $d_1$ we take $d_1(\mu, \nu) = \sup \{|\mu(A) - \nu(A)|: A \in B\}$, where $B$ is the set of Borel sets in the group $X$, while for $d_2$ we take the metric $d_2(\mu, \nu) = \sup \{|\mu(y) - \nu(y)|: y \in X^*\}$, where $\mu$, $\nu$ are the characteristic functions (c.f.) of the distributions $\mu$, $\nu$, respectively; $X^*$ is the group of characters of the group $X$. Our result is the following

**THEOREM.** Assume that $F$ is a completely finite measure on a group $X$ such that $F^{n+1} \perp F^n$ for any natural numbers $n \neq m$. Assume that the distributions $\mu_j, j = 1, 2$, satisfy the inequality

$$\sigma(\mu_1 \times \mu_2, e(F)) < \epsilon, \quad 0 < \epsilon < \epsilon^*$$

Then there exist elements $x_1, x_2 \in X, x_1 + x_2 = 0$, depending only on the distributions $\mu_j$ such that $\mu_j((x_j)) > c_n, j = 1, 2$, and we have the relations $\chi_j(\mu, E_{x_j} \times e(F)) < c_1(\ln \ln (1/e)/\ln (1/e))$, $j = 1, 2$, where $E_{x_j}$ is the distribution concentrated at the point $x_j$, $F_j$ is the restriction of the measure $(\mu_j((x_j)))^{-1}(E_{-x_j} \times \mu_j)$ to the set $S(F)$, where the measure $F$ is concentrated, and $c_0 > 0, c_1 > 0$ are constants depending only on the distribution $e(F)$.

It is easy to see that this theorem contains the above given result of G. M. Fel'd'man.

From this theorem we also obtain the following estimate of the quantity $\beta_{d, a}(\epsilon, e(F))$ where the completely finite measure $F$ satisfies the assumptions of the theorem:

$$\beta_{d, a}(\epsilon, e(F)) < c_{-\epsilon}(\ln \ln (1/e)/\ln (1/e))$$

$c_\epsilon > 0$ being a constant, depending only on the distribution $e(F)$. We mention at once the unimprovability of the estimate (2) in the following sense. Let $X = R^1$ and let $F$ be a measure concentrated at the point $x = 1$. In [6], the author has constructed sequences of distributions on $R^1(\mu_n)_{n=1}^\infty, n = 1, 2$, such that the inequalities

$$\sigma(\mu_{n+1} \times \mu_n, e(F)) < \exp \{-c_n \ln n\}$$

hold, where the constant $c_n > 0$ depends only on $F$ and does not depend on $n$. Moreover, the c.f. of the distributions $\mu_n$ have the form

$$\hat{\mu}_n(t) = \exp \left[ \frac{1}{2} \lambda \left( e^{n\lambda} - 1 \right) + \frac{\delta(n)}{n} \left( e^{x\lambda} - 1 \right) \right], t \in R^1,$$

$\lambda = F(X), \delta(\lambda) > 0$ is a sufficiently small constant, depending only on $\delta$. By virtue of D. A. Raikov's theorem [1], the set of the components $\mu_0$ of the distribution $e(F)$ consists of the distributions with c.f. of the form $\hat{\mu}_0(t) = \exp \{ \lambda_0 \left( e^{\lambda_0} - 1 \right) \}$, $\mu_0 \in R^1, 0 < \lambda_0 < \lambda$. For the c.f. $\hat{\mu}_1(t)$ and $\hat{\mu}_0(t)$ we have the obvious inequalities

$$\inf \sup_{\mu \in K_0} \inf_{\nu \in K_0} |\hat{\mu}_1(t) - \hat{\mu}_0(t)| > c_1 \frac{\lambda_0}{n}, \forall n \in N,$$

where $c_0 > 0$ is a constant, depending only on the measure $F$. The comparison of the estimates (3), (4) leads for the one-dimensional Poisson distribution $e(F)$ to the lower bound $\beta_{d, a}(\epsilon, e(F)) > c_{-\epsilon}(\ln \ln (1/e)/\ln (1/e))$, $c_\epsilon > 0$ being a constant, depending only on the measure $F$.

Proof of the Theorem. We shall carry out the proof by assuming $\epsilon > 0$ sufficiently small: $\epsilon < \epsilon(F)$, which, obviously, does not restrict the generality of our conclusions. In the sequel, positive constants, depending only on the distribution $e(F)$, will be denoted,