STATISTICAL ANALYSIS OF THE AMPLITUDES OF RANDOM STRESSES

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A study is made of the distribution law for the amplitudes of random stresses and associated parameters. A theoretical law is proposed to describe an empirical distribution, and a comparison is made between the deviations of the endurances corresponding to these laws and the coefficient of variation of random endurance values. An empirical formula is obtained to determine maximum stress amplitudes.

Determination of the character of random stresses that are variable over time is necessary not only to analyze the stress state of products, but also to substantiate requirements established for modeling and loading systems and systems designed to record service loads and analyze them on a computer. This problem often reduces to determination of an empirical distribution of stress amplitudes or statistical moments, as well as the approximation of empirical distributions by theoretical distributions. Here, the parameters of the distribution and the agreement between the empirical and theoretical data are usually evaluated solely from a statistical standpoint, with no regard for the mechanism of the load's effect. It is particularly important to be able to use the experimental data to predict the highest values of the stresses that determine the strength of the product.

In examining the distribution law and choosing the associated parameters, we will proceed on the basis of endurance (in cycles) defined by the following formula for loads of variable amplitude

\[ N = \left( \frac{\sigma_{\text{max}}}{N(\sigma)} \right)^{-1}, \]  

where \( f(\sigma) \) is the probability density function of the stress amplitudes; \( N(\sigma) \) is the equation of the fatigue curve in the case of harmonic loading; \( \sigma_{\text{max}} \) is the maximum amplitude of the stresses.

Thus, the permissible error of endurance is a criterion by which we estimate the load parameters and discard some of the amplitudes.

We will assume that it is necessary to determine the range of stress amplitudes that has the greatest effect on endurance in the case of loading in a normal narrow-band process. Let the fatigue curve of the product being loaded have the form

\[ N(\sigma) = N_0(\sigma_0/\sigma)^m, \]  

where \( N_0 \) and \( \sigma_0 \) are base parameters; \( m \) is a constant.

We will assume that the amplitudes of the process are distributed in accordance with Rayleigh's law

\[ f(\sigma) = \frac{\sigma}{S^2} \exp\left(-\sigma^2/(2S^2)\right), \]

where \( S \) is the standard deviation (SDN) of the process.

Allowing for the well-known expression for endurance in the case of narrow-band loading

\[ N_\infty = N_0(\sigma_0/S)^m(2^{m/2}\Gamma(m/2 + 1))^{-1}, \]
we define the damage intensity function as
\[ f_N(\sigma) = \frac{dn}{N(\sigma)d\sigma} = \frac{r^{m/2}}{2^{m/2} \Gamma(m/2 + 1)} \int_{t_1}^{t_2} f(t) dt, \]
where \(dn\) is the number of cycles corresponding to the range of stress amplitudes from \(\sigma\) to \(\sigma + d\sigma\) during service (or testing); \(\Gamma(\cdot)\) is the gamma function.

The area under the curve \(f_N(\sigma)\) is equal to unity. The hatched part of this area (Fig. 1) corresponds to that part of endurance used up during loading in a process with amplitudes bounded by the values \(\sigma_1 = S - t_1\) and \(\sigma_2 = S - t_2\). Here, since we ignore amplitudes with \(\sigma < \sigma_1\) and \(\sigma > \sigma_2\), the error of endurance (in the direction of its increase) is equal to
\[ \delta = \frac{N - N_x}{N} = \left[ \int_{t_1}^{t_2} f(t) dt \right]^{-1} 2^{m/2} \Gamma(m/2 + 1) - 1 = \left[ F(t_2^2, m + 2) - F(t_1^2, m + 2) \right]^{-1} - 1, \]
where \(F(\cdot)\) is the value of the distribution function \(\chi^2\) for quantiles \(t_1, t_2\) and \(m + 2\) degrees of freedom.

For example, for \(m = 8\) and an endurance error \(\delta = 20\%\), we can take \(t_1 = 2.2, t_2 = 4\). These values correspond to an area under the curve \(f(t)\) equal to roughly 0.1, i.e., only approximately 10\% of the amplitudes actually determine the endurance of the product.

It is obvious from this that for a fatigue curve
\[ N(\sigma) = \begin{cases} N_0(\sigma_{-1}/\sigma)^m, & \sigma \geq \sigma_{-1} \\ \infty, & \sigma < \sigma_{-1} \end{cases} \]
with \(t_{-1} = \sigma/S < 1, m > 4\), we can also use Eq. (1) to calculate endurance. In this case, the error of endurance at \(\sigma_{max} \to \infty\) will be \(\delta = F(t_{-1}^2, m + 2)\).

We can assume on the basis of the above example that difficulties might be encountered when statistical methods employing the entire volume of data are used to check the agreement between empirical and theoretical distributions. We will illustrate this by using the \(\chi^2\) criterion as an example:
\[ \chi^2_{r-1,J} = \sum_{i=1}^{r} \left( \nu_i - np_i \right)^2 / (np_i), \]
where \(\chi^2_{r-1,J}\) is a quantity having a \(\chi^2\) distribution with \(r - 1 - J\) degrees of freedom, under the condition that the theoretical distribution be the true distribution and that the number of values in the sample \(n \to \infty\); \(J\) is the number of parameters of the distribution determined for the sample; \(r\) is the number of grouping intervals; \(\nu_i\) is the number of values of the random variable falling within the \(i\)-th interval; \(p_i\) is the corresponding theoretical probability distribution for the given interval.

If the true distribution differs from the theoretical distribution, then the \(\chi^2\) distribution asymptotically satisfies the expression.