ANALYSIS OF DISCRETE AUTOMATIC CONTROL SYSTEMS
WITH VARYING PARAMETERS USING THE METHOD OF
ORTHOGONAL EXPANSIONS. I

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An analysis of discrete automatic control systems with varying parameters is considered
when the coefficients of the difference equation of the system satisfy specific conditions.
The analysis is carried out by expanding the output signal in a specially formulated ortho-
normalized system of functions. The computation algorithm may be realized on a digital
computer.

Let us consider the analysis of linear discrete automatic control systems with varying parameters.
Assume that the dynamics of the discrete automatic control systems can be described by a linear difference
equation of the i-th order with varying coefficients

\[ a_i(n)x(n + i) + a_{i-1}(n)x(n + i - 1) + \ldots + a_0(n)x(n) =
\]

\[ b_i(n)y(n + r) + b_{i-1}(n)y(n + r - 1) + \ldots + b_0(n)y(n), \]

where \( x(n) \) is the output coordinate of the system, \( y(n) \) is the input, and it is assumed that \( i > r \).

Analysis of a discrete automatic control system is understood to mean determination of the response
of the system to an arbitrary input, specifically, for example, an impulse transient function.

This method of investigation is applicable to asymptotically stable discrete systems when the input
\( y(n) \in L^2[0, \infty) \). In this case, if \( y(n) \in L^2[0, \infty) \) (for example, \( y(n) = 1(n) \) or \( y(n) = \sum_{k=0}^{N} a_k n^k \)), then the solution
of the analysis problem first requires the proposed method to be used in order to determine the impulse
transient function of the system, and then, using well-known methods, the output response must be found.

Depending on the character of the variation of the coefficients of Eq. (1), one may consider the fol-
lower particular cases.

The First Case. Assume that the coefficients of the difference equation (1) may be written in the
form

\[ a_i(n) = \sum_{k=0}^{i} A_k e^{-kn} \quad (l = 0, 1, \ldots, l), \]

\[ b_i(n) = \sum_{k=0}^{l} B_k e^{-kn} \quad (l = 0, 1, \ldots, r). \]

Assume that the input \( y = y(n) \) is stipulated.

Let us determine the response of a discrete automatic control system that can be described by the
difference equation (1) whose coefficients satisfy the condition (2). *

*Without loss of generality of the reasoning, we consider the case of zero initial conditions below.
The solution of Eq. (1) shall be found in the form of an expansion in a certain specially chosen system of orthonormalized functions.

Let us assume that the input \( y(n) \) and the output coordinate \( x(n) \) are Laplace-transformable. Let us use the notation

\[
\mathcal{D}\{x(n)\} = X^*(q), \\
\mathcal{D}\{y(n)\} = Y^*(q).
\]

Taking into account the bias theorem in the domain of originals and the shift theorem in the domain of transforms [1], Eqs. (3) may be represented in the following form:

\[
\mathcal{D}\{ A_k e^{-nk} x(n + m) \} = A_k e^{m(q-k)} X^*(q + k), \\
\mathcal{D}\{ y(n + m) \} = B_k e^{m(q-k)} Y^*(q + k).
\]

Equation (1) is written in the following form with allowance for (2):

\[
x(n + l) \sum_{k=0}^{g} A_k e^{-nk} + x(n + l - 1) \sum_{k=0}^{g} A_k e^{-nk} + ... + x(n) \sum_{k=0}^{g} A_k e^{-nk} = y(n + r) \sum_{k=0}^{g} B_k e^{-nk}
\]

\[
x(n + r - 1) \sum_{k=0}^{g} B_k e^{-nk} + ... + y(n) \sum_{k=0}^{g} B_k e^{-nk}.
\]

We apply the Laplace transformation to both sides of Eq. (5). Taking Eq. (4) into account, we shall have

\[
\sum_{k=0}^{g} A_k e^{(q-k)} X^*(q + k) + \sum_{k=0}^{g} A_k e^{(q-k)} X^*(q + k)
\]

\[
+ ... + \sum_{k=0}^{g} B_k e^{(q-k)} Y^*(q + k) \sum_{k=0}^{g} B_k e^{(q-k)} Y^*(q + k)
\]

\[
= e_o(q) Y^*(q) + e_1(q) Y^*(q + 1) + ... + e_g(q) Y^*(q + g),
\]

where

\[
d_k = \sum_{i=0}^{g} A_k e^{(q-i-k)} \quad (k = 0, 1, ..., g),
\]

\[
e_k = \sum_{i=0}^{g} B_k e^{(q-i-k)} \quad (k = 0, 1, ..., g).
\]

If the input \( y(n) \) is stipulated, then the right side of Eq. (7)

\[
e_o(q) Y^*(q) + e_1(q) Y^*(q + 1) + ... + e_g(q) Y^*(q + g) = \Phi(q)
\]

represents a known function of the variable \( q \).

Let us choose the linearly-independent system of functions

\[
f_1(n) = d_0(1) e^{-n} + ... + d_g(1) e^{-n(q+1)},
\]

\[
f_2(n) = d_0(2) e^{-2n} + ... + d_g(2) e^{-n(q+2)},
\]

\[
... ... ... ...
\]

\[
f_g(n) = d_0(g) e^{-gn} + ... + d_g(g) e^{-n(q+g)}.
\]