ON THE EFFECT OF COLLISIONS ON THE
PROPAGATION OF PLASMA WAVES NEAR
THE HARMONICS OF THE ELECTRON
GYROFREQUENCY

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The transverse propagation of plasma waves in a magnetically active plasma near the harmonics of the electron gyrofrequency is considered for the condition that the wavelength is shorter than the gyroradius of an electron. Besides relativistic effects [14], collisions of electrons with other particles are taken into account. The derived dispersion equation is analyzed in the particular cases of weak and strong collisions, and likewise as a function of the resonant frequency detuning.

1. Problems of the kinetic theory of the propagation of high-frequency waves in a plasma perpendicular to the external magnetic field \( H_0 \) have been considered repeatedly (see, for example, the papers [1-4], and the monographs [5-7]). In recent years interest has risen in the transverse propagation of waves in the frequency ranges \( \omega \approx n\omega_H \) (\( \omega_H \) is the gyrofrequency of electrons, \( n = 1, 2, 3, \ldots \)) in connection with the observation of resonances during the probing of the upper atmosphere [8, 9] and during the performance of a series of experiments on laboratory plasma [10, 11]. In a number of papers, whose results are presented in the review [12], a qualitative explanation is given of the resonances observed at the harmonics of the gyrofrequency, which result from the excitation of slow electrostatic waves. Although in these papers an entire series of singularities of such waves was considered, insufficient attention was devoted to the consideration of dissipative processes. Keeping in mind first of all ionospheric applications, it is necessary to indicate the necessity of taking collisions into account in analyzing the behavior of the refractive index near the harmonics of the gyrofrequency.

In connection with what has been said above, a previous paper [13] considered the effect of collisions and relativistic effects on the transverse propagation of extraordinary and plasma waves near the frequencies \( \omega \approx n\omega_H \). Under these conditions it was assumed that \( r_H/\lambda \ll 1 \) (\( r_H \) is the gyroradius of an electron; \( \lambda \) is the wavelength). Whereas for the ordinary and extraordinary waves an analysis of the case \( r_H/\lambda \ll 1 \) is practically sufficient [14] for \( \omega \approx \omega_H \), it follows that for longitudinal waves the reverse condition may also develop. The purpose of the present communication is precisely the approximate consideration of the effect of collisions and relativistic effects on the transverse propagation of plasma waves near the gyroresonances of electrons for \( r_H/\lambda \gg 1 \).

2. The dispersion equation for plasma waves for transverse propagation in the neighborhoods of the frequencies \( \omega \approx n\omega_H \) may be obtained using the method developed in [14]. Assuming that the plasma waves are purely longitudinal and taking account of the collisions of electrons with other particles in the \( \tau \)-approximation [it should be noted that the relaxation model of the collision integral in the given case takes into account the interaction of particles only in a rough approximation. In [16] it was shown that the use of a more rigorous approximation of the collision integral, which allowed the law of conservation of the number of particles to be taken into account, does not lead to substantial differences from the relaxation model when the effect of collisions on the propagation of waves near the gyroresonances is considered. The use of the exact collision integral in the present paper in the form proposed by Landau (as is done, for example, in Sec. 21) presents well-known mathematical difficulties] we arrive at the following dispersion


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relationship:

\[ 1 - \frac{\omega_p^2}{\omega_H^2} \frac{1}{\rho'^2} \int_{\rho'}^{\infty} \frac{\exp \left[ -\mu \delta \left( \frac{\rho}{z} - 1 \right) - i \mu \frac{\nu}{\omega} \left( \frac{\rho}{z} - 1 \right) \right] c^{-\gamma I_n(z)} dz}{z^{1/2}} = 0, \]  

(1)

where

\[ \mu = \frac{m_0 c^2}{T}, \quad \rho = \frac{1}{\mu} \left( \frac{e k}{\omega_H} \right)^2, \quad \delta = 1 - \frac{\eta \omega_H}{\omega}. \]

In Eq. (1) we have used the following notation: \( k \) is the wave vector; \( \omega_p \) is the plasma frequency; \( T \) is the temperature in energy units; \( m_0 \) is the rest mass; \( \nu \) is the electron-collision frequency; \( I_n \) is a modified Bessel function. In the absence of collisions (\( \nu = 0 \)) Eq. (1) goes over into the corresponding expression of [14].

As was indicated above, the analysis will be carried out for \( r_H/\lambda \gg 1 \), i.e., \( |\rho| \gg 1 \). It is likewise assumed that the parameter \( \mu = m_0 c^2 / T \gg 1 \). Since we are interested in waves for which \( \lambda \) is small in comparison with the gyroradius of the electrons, one may use the asymptotic expansion of the functions \( I_n(z) \) for large values of the argument, which leads to the equation

\[ \rho'^2 = \mu \frac{\omega_p^2}{\omega_H^2} \int_{\rho'}^{\infty} \frac{\exp \left[ -\mu \delta \left( \frac{\rho}{z} - 1 \right) - i \mu \frac{\nu}{\omega} \left( \frac{\rho}{z} - 1 \right) \right]}{z^{1/2}} dz, \]

(2)

where \( \rho' \) is a certain quantity beginning with which the expansion indicated above for \( I_n(z) \) is valid. For the condition \( r_H \gg \lambda \) the value of the integral in (1) is determined basically by the quantities \( |z| \gg 1 \). It may be shown that for integration in (1) a large contribution is made by the intervals from \( \rho' \) to \( \rho \) (in comparison with the interval \( 0 < z < \rho' \)). If \( \rho \gg \rho' \) and, in addition \( \ln \rho > \ln \rho' \), then the value of the integral in (2) is determined by the upper limit of \( \rho \) with logarithmic accuracy.

After integration the dispersion equation (2) takes the form

\[ \rho'^2 = \mu \frac{\omega_p^2}{\omega_H^2} \frac{1}{2\pi} \left\{ \text{Ei} \left( -\alpha \frac{\rho'}{\rho} \right) - \text{Ei} \left( -\alpha \right) \right\}, \]

(3)

where \( \text{Ei}(-\alpha \rho'/\rho) \), \( \text{Ei}(-\alpha) \) are integral exponential functions of a complex argument; \( \alpha = \mu \delta + i (\nu / \omega) \mu \).

3. The behavior of the refractive index of plasma waves near the resonances \( \omega \approx n \omega_H \) is evidently determined from Eq. (3) by the function \( f = e^{\alpha} \left\{ \text{Ei}(-\alpha \rho'/\rho) - \text{Ei}(-\alpha) \right\} \). Since it is difficult to trace its dependence on the parameters \( \mu \delta, \mu (\nu / \omega) \) in general form, let us consider a number of particular cases. First we replace \( \text{Ei}(-\alpha \rho'/\rho) \) and \( \text{Ei}(-\alpha) \) by the Kummer function \( U(1; 1; x) \) in the expression for \( f \) by using the well-known relationship between integral exponential and degenerate hypergeometric function [15]:

\[ f = \exp \left( -\alpha \frac{\rho'}{\rho} \right) U \left( 1; 1; \frac{\rho'}{\rho} \right) + U(1; 1; \alpha). \]

(4)

When the inequality

\[ \left| \alpha \frac{\rho'}{\rho} \right| \ll 1 \]

(5)

is fulfilled Eq. (4) can be reduced to the form [15]

\[ f = \exp \left( -\alpha \frac{\rho'}{\rho} \right) \left| \lg \frac{\rho'}{\rho} \right| - \lg \alpha = \lg \frac{\rho}{\rho'}. \]

Assuming the quantity \( \rho' \) to be exponentially small in comparison with \( \rho \), we obtain the estimate

\[ f \approx \lg \rho. \]

Condition (5) may hold for fairly small collisions, so that the inequality \( \mu (\nu / \omega) = m_0 c^2 / T (\nu / \omega) \ll 1 \) is valid, and for a small resonance detuning \( |\mu \delta| \ll 1 \).

For fulfillment of the inequality

\[ |\alpha| \gg 1 \]

(6)