ON THE TRANSFORMATION OF THE STOKES PARAMETERS FOR BACKSCATTERING OF ELECTROMAGNETIC WAVES

B. N. Vvedenskii, E. N. Chernyaev, I. S. Krylov, and S. I. Romanov

Expressions are derived for the interaction matrices which describe the transformation of the Stokes parameters for backscattering of a plane quasimonochromatic electromagnetic wave by an arbitrary statistical object or by an isotropic object. By means of the matrices that have been found the determination of the mutual correlation of the orthogonal components of the scattered field must be considered in an arbitrarily chosen polarization basis for arbitrary elliptic polarization of the incident wave. For the isotropic object the modulus and argument of the generalized correlation coefficient of the linear orthogonal components of the scattered field are obtained.

Introduction

It is well known that the polarization of electromagnetic waves can be described completely by the Stokes vector-parameter \( S \) (I, Q, U, V). For the most general case of a partially polarized wave the components of the vectors \( S \) in a linear basis have the form

\[
\begin{align*}
I &= \bar{E}_x^2 + \bar{E}_y^2, \\
Q &= \bar{E}_x^2 - \bar{E}_y^2, \\
U &= \bar{E}_x^2 + \bar{E}_y^2, \\
V &= -j(\bar{E}_x^2 - \bar{E}_y^2),
\end{align*}
\]

where \( \bar{E}_x, \bar{E}_y \) are the components of the complex field vector (the bar above a quantity denotes time averaging).

For a quasimonochromatic wave these components may be written in statistical notation [1]:

\[
\begin{align*}
I &= 2(D_x + D_y), \\
Q &= 2(D_x - D_y), \\
U &= 4\sqrt{D_x D_y} \rho(0), \\
V &= -4\sqrt{D_x D_y} \rho \left( \frac{\pi}{2\omega} \right),
\end{align*}
\]

where \( D_x, D_y \) are the dispersions of the orthogonal linearly polarized components; \( \rho(0) \) and \( \rho(\pi/2\omega) \) are the values of the normalized correlation function \( \rho(\tau) \) of these components for \( \tau = 0 \) and \( \tau = \pi/2\omega \), respectively,

\[
\rho(\tau) = \frac{E_{x0}(t)E_{y0}(t)\cos(wt + \Phi_{xy}(t))}{\sqrt{|E_{x0}(t)|^2|E_{y0}(t)|^2}},
\]

\( E_{x0}(t), E_{y0}(t) \) are the real amplitudes of the orthogonal components; \( \Phi_{xy}(t) = \Phi_y(t) - \Phi_x(t) \) is the phase difference between the orthogonal components.

As a consequence of the linearity and homogeneity of the Maxwell equations the interaction of radiation with a reflecting object may be described by a linear homogeneous transformation of the vector-parameter [2]:

\[ S_2 = MS_1, \]  

(3)

where \( S_1 \) is the vector-parameter of the incident (\( i = 1 \)) and reflected (\( i = 2 \)) waves; \( M \) is the interaction matrix:

\[
M = \begin{pmatrix}
m_{11} & m_{12} & m_{13} & m_{14} \\
m_{21} & m_{22} & m_{23} & m_{24} \\
m_{31} & m_{32} & m_{33} & m_{34} \\
m_{41} & m_{42} & m_{43} & m_{44}
\end{pmatrix}. \tag{4}
\]

The purpose of the present paper is to establish a relationship between the elements of the interaction matrix and the parameters of the scattered field which may be measured relatively simply in practice. The analysis is carried out for the following constraints: 1) the object is irradiated by a plane electromagnetic wave; 2) the transformation of polarization is considered for reflection only; 3) only those cases of reflection are investigated which can be described by linear homogeneous equations. As an example which illustrates the possibilities of the application of the results obtained, the calculation of the generalized correlation coefficient of the linear orthogonal components of the scattered field is presented for arbitrary elliptical polarization of the radiation.

1. Calculation of the Interaction Matrix

Let us consider a reflecting object in the field of a plane elliptically polarized electromagnetic wave having a complex electric-field vector

\[
E_i = \begin{pmatrix}
E_{ix} \\
E_{iy}
\end{pmatrix} = \begin{pmatrix}
\cos \delta \\
\sin \delta \text{e}^{i \theta}
\end{pmatrix},
\tag{5}
\]

where \( \theta \) is the argument of the phasor; \( \tan \delta = |E_{iy}/E_{ix}| \) is the modulus of the phasor.

The vector \( E_2 \) of the field scattered in the back direction is determined by the known expression

\[
E_2 = \begin{pmatrix}
E_{2x} \\
E_{2y}
\end{pmatrix} = NE_i.
\tag{6}
\]

where \( N = \begin{pmatrix}
a_{xx} & a_{xy} \\
a_{yx} & a_{yy}
\end{pmatrix} \) is the statistical backscattering matrix whose symmetry derives from reciprocity theorem for the case when it may be assumed that the transmitting and receiving antennas are situated at the same point in space [1]. Note that the matrix \( N \) is determined with an accuracy of up to a constant coefficient which depends on the distance between the observer and the scattering object. Let us chose this coefficient to be such that the squares of the moduli of the elements of the matrix \( N \) coincide with the effective scattering area (ESA) of the object for corresponding polarizations.

In order to find the elements of the matrix \( M \) we calculate the Stokes parameters of the scattered field for four polarizations of the radiation: horizontal, vertical, inclined with an angle of orientation \( 45^\circ \), and circular.

a) Horizontal polarization:

\[
E_{1e} = \begin{pmatrix}
1 \\
0
\end{pmatrix}; \quad I_{1h} = Q_{1h} = 1; \quad U_{1h} = V_{1h} = 0.
\tag{7}
\]

From (6) we find

\[
E_{2h} = \begin{pmatrix}
a_{xx} \\
a_{xy}
\end{pmatrix}.
\tag{8}
\]

Substituting (8) into (1) we obtain the Stokes parameters of the scattered wave with allowance for (2):