Relativistic Quantum Mechanics over Stochastic Phase Space

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Stochastic quantum mechanics is a quantum theory in which the basic limitations of real-world measuring instruments, due to their intrinsically quantum nature, are taken into account. Among other things this leads to a new operational definition of space-time, called quantum space-time. Fundamental to this approach is the formulation of quantum mechanics over phase space rather than just over position or momentum space. A concept of extended particle is a natural outgrowth of this development. Gauge and internal symmetry have a natural place within the theory, and preliminary computations combining some old ideas due to Born with more recent ideas on symmetry breaking suggest that the theory could lead to a mass formula compatible with known data on the low-lying baryons.

1. INTRODUCTION

Classical physics is said to be deterministic because the equations which govern the evolution of a classical physical system typically uniquely determine the state of the system at all future times, once the state of the system has been precisely specified at some initial time. As long as one is willing to permit idealized measuring instruments (measuring rods for example, on which perfectly exact and sharp markings are engraved), then the determination of the future state of a system from an initially precise state is not a problem. However, any initial imprecision in the specification of the state of a classical system will result in (generally greater) future imprecision, and as is well known, perfectly precise measuring instruments do not exist in reality. So the result of a realistic measurement of a classical observable should in fact be a value of the observable together with a

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probability distribution, characteristic of the measuring device used, specifying what readings one ought to expect.

Quantum physics, in its conventional formulation, allows that certain pairs of observables may not be simultaneously prescribed or measured with arbitrary accuracy in accordance with Heisenberg's uncertainty principle. But as far as a single observable is concerned, the usual approach permits arbitrarily precise specification of its value. One flaw with this approach is that real instruments are constructed of quantum mechanical and not classical mechanical point particles, and therefore in the limit of the ultimate instrument, namely, a quantum mechanical particle itself, one should not expect to recover arbitrarily high precision. Despite this drawback, however, the imprecisions of a quantum instrument (for example, a particle used as a position marker) are not arbitrary but must be consistent with the uncertainty principle (so that a quantum particle used to measure position and velocity would possess intrinsic but well-defined limitations). The description of quantum mechanics which takes into account these fundamental difficulties has been achieved and has become known as stochastic quantum mechanics (for a review of the entire program see Prugovečki, 1984).

In this paper we give a brief review of the basic notions of the theory of stochastic quantum mechanics together with an account of some recent developments. Section 2 concerns the covariance properties of the stochastic phase space concept which underlies the entire approach. The reciprocity principle of Born and its role within the theory is outlined in Section 3. The introduction of gauge freedom via the canonical commutation relations is described in Section 4, and in Section 5 we give some preliminary calculations leading to a mass formula which agrees well with experiment in the case of low-lying baryons.

2. STOCHASTIC PHASE SPACE

Suppose a simultaneous measurement of position $Q$ and momentum $P$ of a particle is to be carried out. The outcome of such a measurement will not be simply the coordinates $q$ of position and $p$ of momentum, but rather $q$ and $p$ together with confidence functions $\chi_q(x)$ and $\hat{\chi}_p(k)$ representing probability densities that $Q, P$ will have values $x, k$ respectively such that the uncertainties of $Q$ and $P$ (equalling the standard deviations of $\chi_q$ and $\hat{\chi}_p$, respectively) obey the uncertainty principle. In the optimal case, the "spreads" of $\chi_q$ and $\hat{\chi}_p$ will be in inverse proportion. In the limit of perfectly sharp position measurements the confidence function $\chi_q(x)$ goes over into a delta function $\delta(x-q)$ located at $q$, whereas $\hat{\chi}_p(k)$ becomes a