THE BASIS SUPPRESSION METHOD

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Abstract

The Basis Suppression algorithm is a simplex-based procedure which allows the efficient extension of current special structure algorithms to problems of special structure except for a single complicating side variable. A basis free of the complicating variable is maintained in this algorithm. Various properties of the algorithm are presented, including a proof of convergence. Computational effectiveness is discussed and has been verified by using the procedure to solve the maximal concurrent flow problem.

1. Introduction

Many interesting and practical problems in the area of mathematical programming can be formulated in a manner which exhibits special structure. Among the types of special structures that have been investigated are networks, networks with side constraints, multicommodity networks, and problems with generalized upper bounding (GUB) constraints. In this work, we present a technique which allows the extension of these algorithms to a specialized class of problems which does not completely satisfy the special structure requirements.

2. Problem description

The problem in which we are interested may be expressed as a linear programming problem with the following formulation:
minimize $z$

subject to: $Ax + qz = b$

$x \geq 0$

$0 \leq z \leq 1$

with the following properties:

- $z$ is a scalar decision variable (side variable),
- $x$ is an $n \times 1$ vector of decision variables (special structure variables),
- $A$ is a known $m \times n$ matrix of rank $m$ (constraint matrix),
- $q$ is a known $m \times 1$ vector, and
- $b$ is a known $m \times 1$ vector.

We will refer to the above problem (P1) as the Special Structure with Side Variable (SSSV) problem. We are interested in SSSV problems for which the matrix $A$ has a special structure which is violated by including the column $q$.

Various methods have been developed to take advantage of inherent or transformed special structure of linear programming models. In turn, these methods may generally be adapted for the SSSV problem in order to use the special structure efficiently. The primary computational difficulty arises if the side variable is allowed to enter the basis and complicate the special structure, an occurrence which, due to the cost structure of our problem, is bound to happen if standard simplex procedures are utilized. Therefore, we have developed an algorithm which prevents this troublesome occurrence from taking place. We call our algorithm the Basis Suppression algorithm.

The reader may recognize some similarity of the SSSV problem to the well-known parametric linear programming problem (see Bazaraa and Jarvis [3]). This involves the solution of an LP problem whose right-hand side vector is parametrically changed. The SSSV problem is similar to this if the variable $z$ is moved to the right-hand side. The parametric linear programming approach involves determining the range of change of the right-hand side vector for which the current basis yields a feasible and optimal solution. If modification of the right-hand side vector beyond this range is desired, the dual simplex method is used to determine the change of basis necessary to allow this further perturbation while maintaining optimality. However, for many types of special structure, the dual simplex method is not easily implementable or may be time-consuming computationally. The Basis Suppression method allows for all the efficiencies developed for primal simplex operations on special structured problems to be carried over to this problem without major modifications. The computational efficiency of this approach is documented later in this paper.