Mode-Locking and Chaos in a Model of Two Coupled Thermostatically Controlled Radiators

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The paper examines a variety of nonlinear dynamic phenomena arising from the interaction between two thermostatically controlled radiators. Each system operates as a delayed, nonlinear control system and performs self-sustained opening and closing cycles when the slope of the regulator characteristics exceeds a critical value. The coupling is caused by heat exchange between the heated air volumes. As the sensitivity of one of the thermostats is changed, a devil’s staircase of frequency-locked oscillations develops. For higher coupling strengths, mode-converting bifurcations, simultaneous periodic solutions and deterministic chaos can be observed.

1 Introduction

Instabilities in nonlinear systems can give rise to a variety of complex dynamic phenomena, including (i) frequency-locking between two or more oscillatory subsystems, (ii) simultaneous solutions, (iii) mode-converting bifurcations as well as (iv) different forms of deterministic chaos. The study of these phenomena has attracted a rapidly growing interest during the last decade, and examples of chaotic motion have been reported for many different physical, chemical and biological systems [1–9].

In the technical realm, Heiden [10] has discussed the existence of chaos in control systems with delayed feedback. Rubio et al. [11] have investigated chaotic motion in an adaptive control system developed as autopilot for large tankers. Virgin et al. [12] have considered chaotic responses of a floating structure such as, for instance, a semi-submersible oil platform. Grabec [13] has analyzed chaos generated in machine cutting processes, and Onken and Wicke [14] have reported on complex dynamic phenomena in experiments with adiabatic fixed-bed reactors for flue gas oxidation. Chaotic dynamics in forced mechanical systems have been investigated, for instance, by Poddar et al. [15] and by Shaw and Tung [16].
In the present paper we consider frequency-locking, mode-converting bifurcations, and chaotic behavior arising from the interaction between two coupled thermostatically controlled radiators. Each system operates as a negative feedback system with proportional control. A more detailed description must also account for hysteresis and nonlinearities in the regulator characteristics. In addition, the system contains an essential delay associated with the serial coupling of radiator, air and sensor heat capacities. Because of this delay, a thermostatically controlled radiator system may become unstable and perform self-sustained opening and closing cycles, if the slope of the regulator characteristics in the working point exceeds a certain threshold. Also the closing characteristics and the hysteresis of the valve can influence the stability of the system.

Interaction between two such radiators, heating separate air volumes, but with a variable degree of heat exchange, leads to a variety of complex behaviors. As the time constant of one of the thermostats is changed, a devil's staircase of frequency-locked oscillations can be observed. For higher coupling strengths, mode-converting bifurcations and deterministic chaos arise. In this regime, we also find simultaneous solutions, i.e., depending on the initial conditions, different stationary solutions can be obtained for the same parameter values.

Our study is directly inspired by full scale tests of thermostatically controlled valves from a variety of different manufacturers performed by Danfoss, Denmark. Figure 1 shows an example of how such a system, under conditions of normal household installations, can become unstable and start to perform self-sustained oscillations with a period of approximately 1 h. The valve is open for 15–20 min. while the room temperature increases. When the temperature registered by the sensor is high enough, the valve closes, and it remains closed until the room temperature has fallen sufficiently for a new cycle to start. By contrast to immediate expectations, such unstable regulation is not necessarily disadvantageous. Similar on-off regulation is applied in a multitude of technical control systems, including refrigeration and air conditioning systems as well as oil and gas burners in most boiler systems. It is also worth noticing that as the demands to the regulator is increased in terms of a steeper slope, lower hysteresis and better closing ability, the tendency of the system as a whole to become unstable also increases.

2 Description of the System

As illustrated in figure 2, we consider a system in which two air volumes are heated by thermostatically controlled radiators. Both radiators are assumed to perform opening and closing cycles. The two air volumes can be viewed as parts of a single room or as two separate rooms, depending on the value of the coupling parameter $K_{wall}$. This parameter measures the heat flow per unit area and per unit of temperature difference between the air volumes. For each subsystem our model includes the inlet pipe which supplies hot water to the radiator, the thermostatic valve, the radiator, the air volume and a representation of the building confine nt. The outdoor temperature and the ventilation are assumed to be constant. Hence, no external disturbance is included, and the observed complicated dynamical behaviors are fully generated within the system.

To comply with technical modeling practice, each radiator is divided into three subsections which individually exchange heat with the surroundings. Similarly, the