The Unique Description of Kinetic and Hydrodynamic Processes

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(Received January 10, 1992)

The kinetic equation for a description of small-scale (kinetic) and large-scale (hydrodynamic) processes in the Boltzmann gas is proposed. It consists of two dissipative terms: (1) the "collision integral" in the velocity space; (2) the dissipative term of diffusion type in the coordinate space. The difference in the definitions of the "continuous medium" for the kinetic and the hydrodynamic levels of description is taken into account, and the corresponding physically infinitesimal scales for kinetic and hydrodynamic regions are introduced. The maximum Reynolds number is defined. The proposed kinetic approach may be useful for effective description of nonequilibrium processes in chemically active media.

1 The Physical Definition of Continuous Medium

In order to construct the generalized kinetic equation for all scales (kinetic and hydrodynamic), it is necessary to introduce the concrete definition of physically infinitesimal scales for these two levels of description of nonequilibrium processes.

The kinetic description. A rarefied (Boltzmann) gas is characterized by one dimensionless parameter

$$\varepsilon = \frac{n r_0^2}{\lambda^2} \ll 1.$$  \hfill (1.1)

$r_0$ is the diameter of atoms, and $n$ is the mean density of the number of particles. This parameter ought to determine the connection between the infinitesimal scales $\tau_{ph}, l_{ph}$ and the corresponding dissipative parameters $\tau, l$ of the Boltzmann gas.

We denote by $N_{ph}$ the average number of particles in the volume $V_{ph}$ and define a new time scale $\tau_{ph}$ as

$$\tau_{ph} = \frac{\tau}{N_{ph}}, \quad N_{ph} = n V_{ph} \sim n r_0^3.$$  \hfill (1.2)
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\[ t_{\text{ph}} \text{ defined above is the time in which each of the } N_{\text{ph}} \text{ particles has undergone a collision. The definition (1.2) is supplemented by the equation} \]

\[ l_{\text{ph}} = t_{\text{ph}} v_{\text{ph}}. \]  

From (1.2) and (1.3) we find the required expressions [1-3]

\[ \tau_{\text{ph}} \sim \sqrt{\varepsilon} \tau \ll \tau, \quad l_{\text{ph}} \sim \sqrt{\varepsilon} l \ll l, \quad N_{\text{ph}} \sim \frac{1}{\sqrt{\varepsilon}} \gg 1. \]  

We can see now that the conditions imposed on the definition of parameters \( \tau_{\text{ph}}, l_{\text{ph}}, N_{\text{ph}} \) are satisfied for the Boltzmann gas.

The hydrodynamic description. In this case the relaxation time is defined by the characteristic scale \( L \) for the hydrodynamic motion, and by the “diffusion coefficient” \( D \). Thus

\[ \tau_{\text{ph}} \sim \frac{L^2}{D}, \quad \text{and by definition} \quad \tau_{\text{ph}}^{G} = \frac{\tau_{D}}{N_{\text{ph}}^{G}}. \]  

The role of \( D \) is played by the viscosity \( \nu \), the thermal conductivity \( \chi \) and the self-diffusion coefficient \( D \). Here and below \( D = \nu = \chi \). The superscript “\( G \)” reminds us that we are dealing with the Gas-dynamic level of description.

In order to find \( \tau_{\text{ph}}^{G}, l_{\text{ph}}^{G}, N_{\text{ph}}^{G} \), from equations (1.5) it is necessary to find the relation between scales \( \tau_{\text{ph}}^{G}, l_{\text{ph}}^{G}, N_{\text{ph}}^{G} \). We distinguish the following two cases:

1. Slow processes. The scales \( \tau_{\text{ph}}^{G}, l_{\text{ph}}^{G} \) are related by the diffusion law

\[ \tau = \frac{1}{2} \left( \frac{\tau_{D}^{G}}{N_{\text{ph}}^{G}} \right)^2. \]  

In this case we have the following results

\[ l_{\text{ph}}^{G} \sim \frac{L}{N^{1/5}} \ll L, \quad \tau_{\text{ph}}^{G} \sim \frac{\tau_{D}}{N_{\text{ph}}^{G}} \ll \tau_{D}, \quad N_{\text{ph}}^{G} \sim N^{2/5} \gg 1. \]  

Here \( N \sim nL^3 \) is the number of particles in volume \( L^3 \).

2. Fast (acoustic) processes. In this case \( l_{\text{ph}}^{G} \sim \tau_{\text{ph}}^{G} v_{\text{ph}} \) and

\[ l_{\text{ph}}^{G} \sim (\varepsilon Kn)^{1/2} L \ll L, \quad N_{\text{ph}}^{G} \sim \frac{N^{1/4}}{Kn^{3/4}} \gg 1. \]  

Here \( Kn = L^{-1} \) is the Knudsen parameter.

In order to characterize the approximation of “continuous medium” it is natural to use the “physical Knudsen parameter” \( K_{\text{ph}} = \frac{l_{\text{ph}}}{L^{-1}} \). For the kinetic region \( L \sim l \), and therefore

\[ K_{\text{ph}} \sim \frac{l_{\text{ph}}}{l} \sim \sqrt{\varepsilon} \sim \frac{1}{N_{\text{ph}}}. \]