MODELING OLIGOPOLISTIC INTERACTION
by
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ABSTRACT

It is shown that standard versions of overt and tacit collusion under quantity-setting oligopoly are formally equivalent. The two approaches are combined to model collusive behavior when firms expect rivals to react to output changes. The combined model yields a structural equation for firm-level price-cost margins that is consistent with empirical studies using line-of-business data.

1. Introduction

Three distinct themes appeared in the earliest scholarly reactions to Cournot's (1838) analysis of duopoly. Perhaps the most prominent was that error lay in specifying quantity rather than price as the decision variable [Bertrand (1883, p. 503); Fisher (1898, p. 126); Edgeworth (1922)]. The current consensus is that it is useful to model both price-setting and quantity-setting markets.

The two remaining themes, also introduced by Bertrand (1883, p. 503), are stated concisely by Fisher (1898, p. 126):

The fault to be found in [Cournot's] reasoning is in his premise that each individual will act on the assumption that his rival's output is constant, and will strive only to so regulate his own output as to secure the largest profit.

The first criticism is that Cournot analyzed a duopoly in which each duopolist acted in the belief that the other's control variable was fixed. The second criticism is that Cournot had each duopolist maximize its own profit, and only its own profit.

Although economists had been tardy in recognizing the importance of Cournot's work, they were not slow to explore alternative specifications, in attempts to relax one or the other of these

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assumptions. Edgeworth (1881, p. 53) introduced what has come to be the "coefficient of cooperation" approach, and formally explored the implications of partial joint profit maximization. Bowley (1924, p. 38) wrote out first-order conditions for conjectural derivatives in duopoly, specifying a model in which expectations of rivals' reactions are not set to zero by assumption.

Three different ways of modeling conjectural variations have appeared in the literature, and two of these have been widely used. Although equivalent in equilibrium, the alternative approaches to modeling conjectures have different implications for firms' behavior out of equilibrium, and for the specification of tests of market power. The relationship among the three specifications is discussed below.

In what follows, I show that the coefficient of cooperation approach is formally equivalent to one of the common conjectural variation specifications. I also show that the two approaches can be combined, to obtain a model of cooperation with conjectures as to rivals' behavior. The combined model yields predictions for a firm-level price-cost margin equation that are consistent with the results of studies of profitability at the line-of-business level.

2. Conjectural Variations Models

A. Alternative Specifications of Conjectures

Consider an oligopoly of \( n \) firms which supply a standardized product to a market with inverse demand curve

\[
p = a - bQ = a - b(q_1 + q_2 + \ldots + q_n).
\]  

(1)

Let \( c_i \) be the constant marginal and average cost of firm \( i \), and for notational convenience, define

\[
Q_{-i} = Q - q_i
\]  

(2)

as the output of all firms except firm \( i \). Three specifications of firm conjectures have appeared in the literature.

Frisch (1933, p. 252) discusses conjectural elasticities, which in the current accepted notation are

\[
\alpha_i = \frac{\partial \log Q_{-i}}{\partial \log q_i} = \frac{q_i}{Q_{-i}} \frac{dQ_{-i}}{dq_i}.
\]  

(3)