Explicit Preconditioned Iterative Methods for Solving Large Unsymmetric Finite Element Systems

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Abstract — Zusammenfassung

Explicit Preconditioned Iterative Methods for Solving Large Unsymmetric Finite Element Systems. A class of Generalized Approximate Inverse Matrix (GAIM) techniques, based on the concept of LU-sparse factorization procedures, is introduced for computing explicitly approximate inverses of large sparse unsymmetric matrices of irregular structure, without inverting the decomposition factors. Explicit preconditioned iterative methods, in conjunction with modified forms of the GAIM techniques, are presented for solving numerically initial/boundary value problems on multiprocessor systems. Application of the new methods on linear boundary-value problems is discussed and numerical results are given.

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Key words: Approximate inverse matrix techniques, explicit preconditioners, parallel iterative methods, unsymmetric finite element systems, initial/boundary-value problems.

1. Introduction

In recent years research efforts have been directed to many aspects of the production of numerical software, and in particular, on the development of efficient and accurate algorithmic procedures for solving computational problems on available machines (uniprocessor or multiprocessor systems), on techniques for analysing these algorithms and on their implementations.

Current research efforts are now focusing on the development, testing and analysis of new numerical algorithms that are needed to effectively exploit multiprocessor systems. There are several principles involved in the parallel algorithm design, i.e. vectorization, "divide and conquer" partitioning, recursive doubling, designing of complete new parallel algorithms, conversion of implicit
forms to explicit forms. Among these principal parallel techniques that can be classified as fundamental "parallel computational methods", the last one offers substantial advantages on the grounds of exploitation of our previous long sequential computing experience.

The implicit preconditioning methods have been effectively used for the efficient solution of large sparse linear systems of algebraic equations. It should be noted that the implicitness has been introduced in sequential computations because of speed, e.g. numerical solution of ODE's and PDE's. By taking out this implicitness and using the corresponding explicit forms, all the available processors can be effectively used for solving certain classes of problems at the same time.

The preconditioning methods incorporate a high-degree of implicitness in their solution processes due to the sequential nature of the forward-back substitution scheme and therefore this property tends to make them inappropriate for solution on the multiprocessor systems.

The derivation of suitable parallel methods was the main objective for which several forms of approximate inverse of a given matrix, based on appropriate splittings and used in place of the approximate factorization, have been proposed [1, 3, 5, 6, 10, 12, 14, 16]. The main motive for the derivation of the Generalized Approximate Inverse Matrix techniques lies in the fact that they can be efficiently used in conjunction with explicit iterative schemes leading to effective semi-direct solution methods, which possess a high degree of explicitness and therefore are particularly suitable for solving large linear systems on parallel and vector processors.

In Section 2 we introduce the Generalized Approximate Inverse Matrix (GAIM) techniques. These techniques are originated from the Waugh-Dwyer’s algorithmic procedure [19] for inverting a real \((n \times n)\) unsymmetric matrix \(A\) of irregular structure, which can be factorized as \(A = LU\), by determining the elements of \(A^{-1}\) without inverting the decomposition factors. Modified forms of GAIM algorithm lead to a class of optimized approximate inverses particularly effective for the solution of large order linear systems resulting from the Finite Element (FE) discretization of initial/boundary value problems [9]. In Section 3, explicit first and second order semi-direct methods, based on the derived GAIM techniques, are presented. Furthermore in this section, explicit Conjugate Gradient-type methods for the solution of large linear unsymmetric FE systems, are introduced. Finally, in Section 4, the applicability of the new parallel iterative methods on 2D boundary-value problems is discussed and numerical results are given.

2. Generalized Inverse Matrix Methods

Let us consider the boundary-value problem