Short Communications / Kurze Mitteilungen

The Durand-Kerner Method for Trigonometric and Exponential Polynomials

P. Weidner, Jülich

Received May 27, 1987; revised February 1, 1988

Abstract — Zusammenfassung

The Durand-Kerner Method for Trigonometric and Exponential Polynomials. The problem of finding all roots of an exponential or trigonometric equation is reduced to the determination of zeros of algebraic polynomials where the well-known Durand-Kerner algorithm can be applied. This transformation of the problem has the additional advantage that the periodicity of the original functions is eliminated and the choice of starting values is simplified.

AMS Subject Classifications: 65H05.

Key words: Trigonometric polynomial, zeros, Durand-Kerner method.


1. Introduction

In a recent paper Makrelov and Semerdzhiev [6] suggest several methods for the simultaneous finding of all roots of algebraic, trigonometric and exponential polynomials. The method for algebraic polynomials is identical to the well known method named after Durand [3] and Kerner [4], which goes back to Weierstrass and was reinvented by several authors; see [9] for some history. The trigonometric and exponential polynomials can easily be transformed to algebraic polynomials to which the Durand-Kerner algorithm can be applied, making special algorithms superfluous.

2. The Algebraic Case

To approximate in the complex plane the $n$ simple zeros $z_k$ of the polynomial

$$P(z) = z^n + a_{n-1} z^{n-1} + \ldots + a_1 z + a_0$$

...
with complex coefficients $a_j$, Durand [3] suggested the iteration process

$$z_{k+1}^{(r)} = z_k^{(r)} - \frac{P(z_k^{(r)})}{\prod_{j=1}^n (z_k^{(r)} - z_j^{(r)})} \quad k = 1, 2, \ldots, n$$

with starting values $z_0^{(0)}, z_1^{(0)}, \ldots, z_n^{(0)}$.


In the paper [8], a whole class of similar methods with high convergence orders is constructed, but at the price of more function evaluations per iteration step. With the original Durand-Kerner algorithm, the roots of the example given there are found after 15 iterations to 10 decimal places if one starts on a circle with radius 21 around the origin. We mention that the example in [8] must be corrected to

$$p(z) = z^{10} - 20 (1 + i) z^9 + 400 i z^7 + 3 \cdot 10^4 z^6$$
$$- 6 \cdot 10^5 (1 + i) z^5 + 12 \cdot 10^6 i z^4$$
$$- 4 \cdot 10^8 z^2 + 8 \cdot 10^9 (1 + i) z - 16 \cdot 10^{10} i.$$

3. The Trigonometric Case

Using Euler's formulas for cos and sin and the substitution $e^{iz} = w$, the problem of finding the $2n$ simple (complex) roots of the trigonometric polynomial

$$T_n(z) = a_0 + \sum_{k=1}^n (a_k \cos k z + b_k \sin k z)$$

in the strip $-\pi < \Re z \leq \pi$ can be reduced to the root finding for an algebraic polynomial of degree $2n$.

With:

$$c_j = \frac{a_{n-j} + i b_{n-j}}{(a_n - i b_n)} \quad j = 0, \ldots, n-1$$

$$c_n = \frac{2a_0}{(a_n - i b_n)}$$

$$c_{n+j} = \frac{a_j - i b_j}{(a_n - i b_n)} \quad j = 1, \ldots, n$$

the resulting algebraic polynomial has the form

$$\hat{T}_n(w) = w^{2n} + c_{2n-1} w^{2n-1} + \ldots + c_1 w + c_0$$