ANALYSIS OF RELAXATIONS FOR THE MULTI-ITEM CAPACITATED LOT-SIZING PROBLEM

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Abstract

The multi-item capacitated lot-sizing problem consists of determining the magnitude and the timing of some operations of durable results for several items in a finite number of processing periods so as to satisfy a known demand in each period. We show that the problem is strongly NP-hard. To explain why one of the most popular among exact and approximate solution methods uses a Lagrangian relaxation of the capacity constraints, we compare this approach with every alternate relaxation of the classical formulation of the problem, to show that it is the most precise in a rigorous sense. The linear relaxation of a shortest path formulation of the same problem has the same value, and one of its Lagrangian relaxations is even more accurate. It is comforting to note that well-known relaxation algorithms based on the traditional formulation can be directly used to solve the shortest path formulation efficiently, and can be further enhanced by new algorithms for the uncapacitated lot-sizing problem. An extensive computational comparison between linear programming, column generation and subgradient optimization exhibits this efficiency, with a surprisingly good performance of column generation. We pinpoint the importance of the data characteristics for an empirical classification of problem difficulty and show that most real-world problems are easier to solve than their randomly generated counterparts because of the presence of initial inventories and their large number of items.

1. Introduction

The capacitated dynamic lot-sizing problem, most easily defined in terms of production planning, consists of determining the quantity and the timing of production for several products in a finite number of periods, so as to satisfy a known demand in each period and minimize the sum of the set-up, production and inventory costs without incurring backlogs. A production capacity is imposed in each period. A set-up cost and a linear cost of production are specified, and the inventory cost is proportional to the quantity and time carried. The costs may vary for each product and each period.

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The problem is known to be NP-hard even for one item [25], and the thrust of recent research has been toward the development of heuristics [44,19,20]; their major advantage is that they provide feasible solutions in relatively short time. Concurrently, various optimal solution methods have been proposed; their basic methodology is to seek a lower bound on the value of the problem as a fathoming device in an optimal enumerative search. Such a lower bound has been calculated in past research via column generation [21], subgradient optimization [53,55], cutting-planes [8] and variables redefinition [22].

Admittedly, finding an optimal solution is a computer-intensive task. It is often argued that in most cases, the approximation of the data does not justify refinement beyond fast heuristics; however, Geoffrion [33] points out the intrinsic value of an exact solution even for approximate data, because most managerial decisions entail the comparison of various alternatives evaluated in several computations: "Not only does an optimizing capability enhance the value of most individual runs, but it also provides the opportunity to make valid comparisons between the results of different runs. This is extremely important because the conclusions reached by the planning project typically rely far more heavily on comparisons between computer runs than on runs considered individually. With "quasi-optimizing" programs, such as so-called cost calculators or simulators fitted with heuristics, one never knows whether different results are due to different inputs or to the vagaries of the computer program." In discrete optimization, it is well-known that the speed of enumeration hinges critically on the quality and the ease of computation of lower bounds, and this remark suffices to justify a quest for good relaxations; moreover, seeking a lower bound presents benefits in its own right:

- By itself, a value furnished by a heuristic says nothing about how much effort should be devoted to improving the solution obtained. On the other hand, bracketing the optimal value from above (by the value of the feasible solution) and below (by the value of the relaxed solution) gives the decision-maker some benchmark for deciding whether to continue the heuristic search or not.

- Occasionally, a lower bound is more important than a feasible solution: for example, in a litigation, a contractor claims that the prespecified cost of performing a service has been dramatically increased by some changes of specifications occurring after drawing up the contract. In this case, a feasible but not optimal cost will not absolve the defending contractor, who in all rigor must provide an optimistic bound on the cost of satisfying the modified contract and then argue that the bound is still above the contracted amount. Further consideration shows that this situation is much more common than it first seems, because similar but generally less formal debates preside over the design of many production plans.