Section II

Single-Machine Scheduling
REDUCIBILITY AMONG SINGLE MACHINE WEIGHTED COMPLETION TIME SCHEDULING PROBLEMS

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Abstract

Various sum of weighted completion time problems are compared. The constraints considered include release date, deadline, and continuous machine processing. Relations between the problems are developed by examining the computational complexity of transforming one problem class into another. These results give indications of the relative computational effort required to solve different problem classes.

Keywords

Weighted completion time, scheduling, complexity.

1. Introduction

Developments in the theory of computational complexity have shown that many scheduling problems belong to the class of NP-complete problems (see Lageweg et al. [16] for a summary). From a practical standpoint, some NP-complete problems are easier to solve than others. Some have pseudo-polynomial time algorithms, while others are NP-complete even with unary encoding (see Lenstra et al. [17]). Some scheduling problems, such as the total tardiness problem, have been solved with 100 jobs in reasonable time by Potts and Van Wassenhove [22]. Other problems have proved intractable even with a very small number of jobs.

Sometimes, seemingly related NP-complete problems have significantly different structures. Consider the one machine weighted completion time problem. The problem with release dates and the one with deadlines are substantially different. With release dates, all job permutations yield feasible solutions. Also, an optimal solution may have idle time between the processing of jobs and there are cases when preemption will improve the solution. The release date problem is still NP-complete when all jobs have unit weights. None of these statements are true with deadline constraints.

Distinct literatures have emerged for these two problems. Papers discussing solution methods for the release date problem include Rinnooy Kan [25], Rinaldi and Sassano [24], Bianco and Ricciardelli [7], Hariri and Potts [15], Belouadah [4, 5], Belouadah et al. [6], and Dyer and Wolsey [11]. Chandra [9], and Dessouky and Deogun [10] examine procedures for the case of equal weights. For the deadline