MEASUREMENTS OF THE LOG-IRRADIANCE DISTRIBUTION OF A LASER WAVE PROPAGATED THROUGH THE TURBULENT ATMOSPHERE

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Abstract. Log-irradiance fluctuations of He-Ne laser light were simultaneously measured at three different points in the receiving plane. The cumulative probability distributions of log-irradiance show excellent agreement with the Rice-Nakagami distribution in the region of weak fluctuations. The irradiance distributions strongly depend upon the locations of the observing points with respect to the center of the beam, as expected from recent theory.

1. Introduction

Irradiance fluctuations of an optical wave are caused when the wave is transmitted through a turbulent atmosphere, and experimental determinations of the distribution function of the log-irradiance appear to be close to the normal distribution in the region of weak fluctuations (Barabanenkov et al., 1971). Also normal-like distributions are reported (Ochs and Lawrence, 1969), even in the region of strong fluctuations where 'saturation' of the variance of irradiance was expected to begin. Nevertheless these facts have not been explained theoretically (Barabanenkov et al., 1971), because Gaussian-type characteristic functions with respect to the field intensity or the logarithm of the intensity never satisfy the equation written in the functional representation. Therefore it is concluded that the irradiance or the log-irradiance distribution cannot follow a normal distribution. In the case where the fluctuations of the medium are small enough, however, the distribution of the fluctuations must be normal in view of the central limit theorem and, therefore, the field intensity becomes a linear function of the medium fluctuations within the framework of perturbation theory. The irradiance distributions may be considered to be approximately normal only in the region of weak fluctuations.

Recently, by considering multiple scattering effects completely, Furutsu (1972) obtained an exact irradiance distribution function for a Gaussian beam with a circular cross-section. The distribution function for this case for the log-irradiance fluctuations is the Rice-Nakagami distribution. For the Gaussian beam with an elliptical cross-section an integral representation may be found for the distribution function (Furutsu and Furuhamza, 1973).

In this paper the results of the comparison between theory and experiments using a He-Ne laser are reported. The data were taken in the region of weak fluctuations and showed excellent agreement with the theoretical distributions.
2. The Rice-Nakagami Distribution with Respect to Log-Irradiance

Furutsu (1972) obtained an exact irradiance distribution function for a collimated beam of Gaussian cross-section by using the parabolic wave equation and also assuming a random medium with a Kolmogorov spectrum. The results are that the distribution \( P(E) \) of the log-irradiance \( E \), is the Rice-Nakagami distribution. If we adopt a cylindrical coordinate system, the \( z \)-axis being taken in the direction of wave propagation, and let \( I_{fs} \) be the irradiance in free space, then

\[
I_{fs}(r, \theta, z) = I_0 f^{-2} \exp \left[ - \left( \frac{r}{bf} \right)^2 \right],
\]

where \( I_0, b \) are constants. We define \( E \), the logarithm of the actual intensity \( I \) by

\[
E = \log \left( \frac{I f^2}{I_0} \right),
\]

where

\[
E_0 = \left( \frac{r}{bf} \right)^2,
\]

\[
\sigma_E = \frac{4}{3} \cdot \alpha k z^3 \left( bf \right)^{-2},
\]

(\( \alpha \) is a constant related to the fluctuation of the atmospheric refractive index and \( R \) is a wave number). The log-irradiance distribution function is given by

\[
P(E) = \sigma_E^{-1} \exp \left[ - \frac{(E - E_0)}{\sigma_E} \right] I_0 \left( \frac{2}{\pi} \right) \exp \left[ - \frac{E_0}{\sigma_E} \right], \quad E < 0.
\]

From (6), the lower-order moments are

\[
M_1 \equiv \langle E \rangle = - \left( E_0 + \sigma_E \right),
\]

\[
M_2 \equiv \langle (E - \langle E \rangle)^2 \rangle = \sigma_E^2 \left( 2E_0 + \sigma_E \right),
\]

\[
M_3 \equiv \langle (E - \langle E \rangle)^3 \rangle = - 2\sigma_E^3 \left( 3E_0 + \sigma_E \right).
\]

The \( m \)th order moment with respect to \( I \) is

\[
\langle I^m \rangle = \left( I_0 f^{-2} \right)^m \left( 1 + m\sigma_E \right)^{-1} \exp \left[ - mE_0/(1 + m\sigma_E) \right].
\]

For \( m = 1 \), Equation (11) is

\[
\langle I \rangle = I_0 b^2 B^{-2} \exp \left[ - (r/B)^2 \right],
\]

where

\[
B \equiv bf \sqrt{1 + \sigma_E}.
\]

In the case of \( \sigma_E < 1 \), the beam width at the receiving point is mainly determined by the