A SIMPLE PROBLEM OF RADIATIVE TRANSFER BY MULTILEVEL ATOMS

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We consider the problem of determining the radiation fields reflected and transmitted by a slab containing multilevel hydrogen atoms and illuminated on one side by a given radiation field. We treat the extreme non-LTE situation in which the populations of the different levels are determined by the radiative processes. We take into account the population and the transfer effects in a self-consistent way by solving the transfer equations in all the lines and continua together with the equations of statistical equilibrium for all levels. We limit ourselves to the idealistic case of rectangular profiles in the lines and continua and to a model of atoms with 4 levels and a continuum. Under conditions close to thermodynamic equilibrium we empirically derive a Schuster-like law for the continua with transmitted radiation fields varying as the inverse of the optical thickness. Turning to out-of-equilibrium conditions we emphasize the crucial role of the loss probability of the Lyα photons. Owing to the rapid decrease of the excitation/ionization degree in the medium and contrary to the conservative case the optical thicknesses of the subordinate transitions now remain finite even when the population of the fundamental level along the line-of-sight becomes infinite. As a result of this relative transparency the strong emission lines formed by recombination mechanisms can escape from the medium. Although the present problem remains largely academic because of the number of simplifications introduced we suggest some possible applications and developments.

1. Introduction. Although very complex transfer problems have been solved successfully the physical understanding of the results rather rests on the comprehension of idealized simpler exercises which can be discussed thoroughly and which display more basic effects. For instance the influence of the scattering in particular cases will often be clarified on the basis of Schuster’s [1] model. The study of the radiative transfer problem in a two-level atom for a homogeneous medium (e.g., Avrett & Hummer [2]; Hummer [3]; Ivanov [4]) has thrown light on the non-LTE effects that may appear under other more realistic specific conditions. The model of planetary nebulae by Menzel [5] and his collaborator in the well-known limiting cases A or B still remains the basic scheme for understanding subsequent studies. Milne’s problem constitutes a powerful tool for partly explaining the radiative mechanisms within a stellar atmosphere and the characteristics of the emerging radiation field.

The present work is devoted to a generalization of Schuster’s problem that is straightforward in its principle yet much more difficult to solve. Whereas Schuster was concerned by scattering at a single frequency we generalize the problem to a multifrequency radiation field and to multilevel atoms. So we consider a slab containing multilevel atoms — say hydrogen atoms — and allow all the possible corresponding radiative transitions to take place according to the usual quantum probabilities. The slab is illuminated on one side by a given radiation field at all relevant frequencies in the lines and continua. We ask for the radiation fields reflected and transmitted by such a medium. We stick to the rule of the game consisting in coupling the population and the transfer effects in a fully self-consistent way. In other words we simultaneously solve the equations of statistical equilibrium for all levels and the transfer equations at all frequencies: the equations of statistical equilibrium express the dependence of the level populations upon the radiative intensities whereas the transfer equations tell us how the radiation fields react to the level populations via the absorption and emission coefficients.
To our knowledge this archetype problem of multifrequency scattering posed in that manner has never been examined before except by Nikogosyan [6] with analytical means in a three-level atom for a semi-infinite medium. Of course the problem is now too difficult to be solved analytically and we have to resort to numerical procedures. In spite of that fact we think that the exercise is interesting because it contains the essence of the radiative mechanisms occurring in a medium in which the LTE assumption is not valid and may thus lead to results that are general enough to elucidate more complex situations. On the other hand our schematic model can in no way supersede realistic calculations which include many more factors than the limited number of parameters used here.

The specifications of the model will be given in Sec. 2. When losses of photons are ignored we arrive at the pure scattering problem for multilevel atoms, which is examined as a starting point in Sec. 3. In Sec. 4 we shift to nonconservative cases and show the dramatic role of the losses of Lyα photons. In Sec. 5 we examine the influence of some parameters upon the radiation fields. In outlining the main results we conclude in Sec. 6 about the relevance of the present model to astrophysical objects.

2. Description of the Model. As we will later allude to, the extended atmospheres of some stars illuminated by the radiation coming from the inner parts we may think of in the present model as a schematic representation of a hydrogen ionization zone from the H II region up to the adjacent H I region and will lead the discussion according to this terminology.

We take a hydrogen atom with 4 levels and a continuum. In principle the electronic temperature should be determined by the equation expressing thermal equilibrium. However, in our idealized model the temperature does not really count because the collisions are not explicitly taken into account and the temperature only fixes the coefficients of recombination from the continuum. In practice the temperature of most H II regions as determined observationally or theoretically fall in a relatively narrow range of values around the “standard” 10,000 K figure. Therefore, we may safely take a value of that order. Of course this value does not imply that the real electronic temperature in the H I outer region is so high. But in our model the thermal temperature of the H I does not play any role.

The profiles of the absorption and emission coefficients in the 6 lines (Lyα, Lyβ, LYγ, Hα, Hβ, and Paα) are taken as rectangular with a total width corresponding to velocities in the range (-v, +v), where v is given either by the thermal value \( \sqrt{2kT_e/m_H} \) or by a convenient value characterizing both the thermal and the random macroscopic velocity fields. This assumption is clearly unrealistic as the random walk of the photons through the medium heavily depends upon the line profile and the presence of "Doppler wings" at the edge of the core rules out the use of a constant cross section for scattering. However beyond the simple Schuster monochromatic problem this modeling assumption allows us to explore the effects of including the transitions between several levels without adding at the same time the redistribution mechanisms within a given line. A more realistic profile with Doppler and damping wings will be considered in coming works.

Likewise we treat each continuum at a single frequency or, more exactly, as if it extended over a certain frequency interval with a constant absorption and emission coefficient. That hypothesis is also quite severe — but certainly less severe than in the case of the lines. We define the radiative coefficients of a mean equivalent continuum as follows, with the Lyman continuum taken as an example.

We start from the number of radiative recombinations per unit volume per unit time onto Level 1 written as \( N_e^2 \beta_1 \), where \( N_e \) is the electronic density and the coefficient \( \beta_1 \) is regarded as known. We introduce a would-be Einstein coefficient \( B_{15} \) defined in such a way that the number of photoionizations (cm\(^{-3}\)sec\(^{-1}\)) is \( N_1 B_{15} J_{15} \), where \( J_{15} \) is the mean radiative intensity at the frequency \( \nu_{15} = \nu_1 \) of the Lyman limit. Neglecting the induced emissions, we would get for the LTE population \( N_1^* \) the equilibrium relation

\[
N_1^* B_{15} J_{15} = N_e^2 \beta_1 ,
\]

where

\[
J_{15} = \frac{2h \nu_1^3/c^2}{e^{h \nu_1/kT}} ,
\]

\[
\frac{N_e^2}{N_1^*} = \frac{2}{g_1} \left( \frac{2\pi m_e kT}{h^2} \right)^{3/2} e^{\frac{-h \nu_1}{kT}} .
\]

We thus find \( B_{15} \) in terms of \( \beta_1 \) as

\[
B_{15} = \frac{2h \nu_1^3/c^2}{(2/g_1)} \left( \frac{2\pi m_e kT}{h^2} \right)^{3/2} \beta_1 .
\]