THE EFFECT OF A CONSTANT EFFLUX ON SOLUTE MOVEMENT TO A ROOT*

by JOHN H. CUSHMAN**

KEY WORDS
Concentration Convection Diffusion Efflux Exact solutions Nutrient Plant root

ABSTRACT
This paper presents two analytical solutions to solute uptake and release by a root (the root exhibits efflux) when both mass and diffusive flows are considered and the water content of the soil is maintained constant. The first solution is for a constant solute concentration at some finite distance from the root, while the second is when there is no solute recharge from the soil outside a cylinder coaxle with the root.

Several examples are presented to show the effect of the effluxive term. It is shown that the point where the ratio of efflux to initial ion concentration in solution is equal to the difference between root absorbing power and water flux at the root, is critical to nutrient depletion or buildup at the root surface. It is also shown for the case of no recharge that the concentration profiles when efflux is considered are markedly different than where efflux is not considered.

INTRODUCTION
The purpose of this article is to examine the effect of efflux (E) on the ion uptake at a root. Under appropriate assumptions the differential equation representing mass-flow and diffusion to a root is

\[
\frac{\partial C}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} \left( r D \frac{\partial C}{\partial r} + \frac{v_0 r_0}{b} C \right)
\]

(1)

where r is the radial distance from the root axis, r_0 is the root radius, C is the ion concentration in solution, D is the diffusion coefficient (considered constant) in the soil, v_0 is the inward flux of water at the root, b is the differential buffer power, and t is the time of uptake.

Equation 1 is subject to two boundary conditions (B.C.) and one initial condition (I.C.). The B.C. can take many forms, of which we will examine only

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** Assistant Professor of Soil Physics, Purdue University.
two. The I.C. can also take on many forms, however we will restrict our attention to the case:

\[ t = 0, \quad C = C_i \]  

where \( C_i \) is some constant initial ion concentration.

The appropriate boundary condition at the root surface is

\[ \frac{D_b}{\partial r} \frac{\partial C}{\partial r} + v_0 C = \frac{kC}{1 + kC/J_{\text{max}}} - E, \quad r = r_0, \quad t > 0, \]  

where \( J_{\text{max}} \) is the rate of ion influx at infinitely high concentration, and \( k \) is the root absorbing power. If we assume the ion concentration in solution is initially low then Eq. 3 reduces (approximately) to

\[ \frac{D_b}{\partial r} \frac{\partial C}{\partial r} + v_0 C = kC - E, \quad r = r_0, \quad t > 0. \]  

Eq. 4 is the inner boundary condition we will use.

If we assume there is no inter-root competition for the nutrient in question then the appropriate outer boundary condition is

\[ C = C_i, \quad r = r_1, \quad t > 0. \]  

If we assume there is inter-root competition for the nutrient (but no competition for water) we can imagine a cylinder about the root (coaxial with the root), of radius \( r_1 \), through which no nutrient flux passes yet through which water will pass. In this situation the appropriate outer boundary condition is

\[ \frac{D_b}{\partial r} \frac{\partial C}{\partial r} + v_1 C = 0, \quad r = r_1, \quad t > 0. \]  

Henceforth we will consider two problems:

Case 1: No inter-root competition.

The governing equations are 1, 2, 4 and 5.

Case 2: Inter-root competition.

The governing equations are 1, 2, 4 and 6.

DIMENSIONALITY

In this section we will introduce dimensionless variables to reduce cases 1 and 2 to a more tractable and useful form.

The dimensionless variables we use are: