A mean-absolute deviation-skewness portfolio optimization model

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It is assumed in the standard portfolio analysis that an investor is risk averse and that his utility is a function of the mean and variance of the rate of the return of the portfolio or can be approximated as such. It turns out, however, that the third moment (skewness) plays an important role if the distribution of the rate of return of assets is asymmetric around the mean. In particular, an investor would prefer a portfolio with larger third moment if the mean and variance are the same. In this paper, we propose a practical scheme to obtain a portfolio with a large third moment under the constraints on the first and second moment. The problem we need to solve is a linear programming problem, so that a large scale model can be optimized without difficulty. It is demonstrated that this model generates a portfolio with a large third moment very quickly.

1. Introduction

Since Markowitz [12], the mean-variance (MV) model has been playing a crucial role in financial optimization. However, large scale MV models consisting of more than a few thousand assets were never solved until recently due to their immense computational difficulty. The recent progress in algorithmic research brought about a dramatic change in the practical role of MV models in constructing a large scale portfolio. In fact, we are now able to solve MV models with more than several thousand assets on a real time basis by using a compact factorization and an interior point algorithm for quadratic programming problems [19]. Also, the

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associated efficient frontier can be generated in a matter of minutes by our code based upon a compact factorization and piecewise linear approximation (see [8] and [13]). Thus, investors can now use MV models as a practical tool for constructing a portfolio on a day to day basis without resorting to classical divide-and-conquer strategies such as asset allocation.

Now that a very large scale MV model is within our reach for the first time after 40 years since its birth, we are ready to prepare for the new era of financial engineering by incorporating new features into MV models or by adding more flexibility to meet the diverse requests of investors. Among such efforts are the inclusion of transaction/illiquidity costs [15] and minimal transaction unit constraints for investors with a smaller fund [8].

This paper is another effort in this direction. Here we will explicitly take into account the third moment in addition to the first and second moments of the rate of return of the portfolio. As noted by Samuelson [16] in the late 50's, the third moment has a very important implication in portfolio analysis when neither of the following two assumptions are satisfied.

(i) The rate of return of the assets follows a multivariate normal distribution.

(ii) Utility of an investor is a function of the mean and variance of the rate of return or can be approximated by a quadratic function of the rate of return.

Unfortunately, however, Kariya et al. [6] showed that the rate of return of the majority of stocks included in the Nikkei 225 Index do not satisfy the normality assumption. Also, we calculated the skewness \(^1\) (the third moment) of the monthly rate of return of 1118 stocks in the Tokyo Stock Exchange to find that the skewness of more than 1000 stocks is positive and that 648 stocks have large skewness which would reject the normality hypothesis at 5% significance level (see table 1).

Also, there are persistent objections against assumption (ii) among practitioners. To see this, let us consider two portfolios \(P_1\) and \(P_2\) whose distributions of the rate of return are shown in fig. 1. Both of these distributions have the same mean and the same standard deviation. Thus \(P_1\) and \(P_2\) are indifferent

\(^1\)Skewness \(\kappa(R)\) of a random variable \(R\) is given by \(\kappa(R) = E[(R - E[R])^3]/E[(R - E[R])^2]^{3/2}\). When \(R\) is normal, then \(\kappa(R) = 0\).