It is apparent that \( \Delta \) is always positive, i.e., the work performed by the plasma against the force of the magnetic field is positive on the average and energy is transferred from the plasma to the electrical lines. If it is possible to operate under conditions such that \( \frac{\chi}{2} (1 - \alpha_0) \) is larger than 2-3 (this is possible if very strong magnetic fields are used) the excess energy per cycle is only weakly dependent on the other parameters and, in order-of-magnitude terms, is equal to the average energy of the plasma. The electrical energy obtained per unit time is proportional to the frequency. When \( \chi \ll 1 \), the power is independent of frequency.

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EQUILIBRIUM DISTRIBUTION OF CURRENT DENSITY IN LINEAR HIGH-CURRENT DISCHARGES

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In the present paper we derive the equilibrium distribution of the electronic and ionic densities in a linear discharge when there is an additional current which flows along the axis of the chamber.

We assume that in the chamber, which is formed by two coaxial cylinders of radii \( r_1 \) and \( r_2 \) (\( r_1 < r_2 \)), there flows a current \( J_1 \) there is an additional conductor which carries current \( I \) along the axis of the chamber. We denote by \( p_e(r) \) and \( p_i(r) \) the electron and ion densities respectively, the symbols \( T_e \) and \( T_i \) denote the respective temperatures and \( B_e \) and \( B_i \) denote the ratio of the longitudinal drift velocities to the velocity of light.* Then, if it is assumed that the pressure tensor is isotropic and that the ions are singly charged, the equations which describe the stationary distribution assume the form:**

\[
\begin{align*}
\rho_e [E_r - \beta_e H_\phi] + \frac{k}{e} \frac{\partial}{\partial r} (\rho_e T_e) &= 0; \\
\rho_i [E_r - \beta_i H_\phi] - \frac{k}{e} \frac{\partial}{\partial r} (\rho_i T_i) &= 0; \\
\frac{1}{r} \frac{\partial}{\partial r} rE_r &= 4\pi \sigma (\rho_i - \rho_e); \\
\frac{1}{r} \frac{\partial}{\partial r} rH_\phi &= 4\pi \sigma (\rho_i \beta_i - \rho_e \beta_e),
\end{align*}
\]

where \( k \) is the Boltzmann constant and \( e \) is the absolute value of the electron charge.

We assume that the drift velocities \( \beta_e \) and \( \beta_i \) and the temperatures \( T_e \) and \( T_i \) are independent of radius; this condition is generally satisfied at fairly low densities. It can then be shown that if one considers an axial charge

* We may note that in all the formulas by \( \beta_e \) and \( \beta_i \) is meant the algebraic value of the velocity, i.e., if it is assumed that \( J > 0 \), then \( \beta_i > 0 \) while \( \beta_e < 0 \).

** We neglect processes which take place at the electrodes.
\[ q = \frac{I}{c} \frac{T_e \beta_i T_i \beta_0}{T_i - T_e} \tag{3} \]

the nonvanishing solutions of Eqs. (1) and (2) are:

\[
\begin{align*}
\rho_e (r) &= \frac{\rho_0}{1 + \xi (p+2) \frac{r}{(p+2)T e \beta_0}} \tag{4a} \\
\rho_i (r) &= \frac{\rho_0}{1 + \xi (p+2) \frac{r}{(p+2)T e \beta_0}} \tag{4b}
\end{align*}
\]

where

\[
\begin{align*}
\rho_0 &= \frac{k}{2\pi e^2} \frac{(p+2) T e (1 - \beta_i^2) + T_i (1 - \beta_i^2)}{(\beta_i - \beta_0)^2} \tag{5a} \\
\rho_i &= \frac{k}{2\pi e^2} \frac{(p+2) T e (1 - \beta_i^2) + T_i (1 - \beta_i^2)}{(\beta_i - \beta_0)^2} \tag{5b}
\end{align*}
\]

\[ P = \frac{4}{\pi - 1} \left[ 1 - \frac{J_0 (1 + x)}{J_0 (1 + x)} \right] \tag{6} \]

\[ x = (z - 1) (1 - 1) + V(z - 1)^2 (1 - 1)^2 + 4z \tag{7} \]

\[ z = \left( \frac{r_i}{r_e} \right)^{p+2} \tag{7a} \]

Thus, the introduction of an additional axial current \( I \) means that the stationary state is possible only if there is a charge \( q \) on the axis; this charge vanishes only if \( I = 0 \) or \( B_e = T e \). It follows from Eqs. (4) and (5) that the gaseous column is charged as a whole and that the charge per unit length is

\[ Q = \frac{J}{c} \frac{T_e \beta_i + T_e \beta_0}{T_e - T_i} = q \frac{J}{I} \tag{8} \]

From Eqs. (4) and (5), we obtain the expression for the current density

\[ j (r) = \frac{J_0}{\pi} \frac{(p+2)^2}{4\xi} \frac{r^{p+2}}{(1 + \xi (p+2) \frac{r}{(p+2)T e \beta_0})^{p+2}} \tag{9} \]

Whence it follows that the nature of the current distribution over the cross section changes as the discharge current is increased.

The current density reaches a maximum value when \( r = r_m \), where

\[ r_m = r_i \left( \frac{p}{p + 2} \right)^{\frac{1}{p + 2}} \tag{10} \]

and

\[ j (r_m) = \frac{J_0}{\pi} \frac{p (p+4)}{16r_m^p} \tag{11} \]

It is of interest to consider some cases in detail:

1) \( I = 0 \).