A SIMPLE PROCEDURE FOR REARRANGING MATRICES*

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Guttman's scalogram board technique for reordering the columns and rows of a matrix is described and its disadvantages are pointed out. A simple and inexpensive procedure for doing the same job without these disadvantages is outlined.

There are a number of problems in psychometrics for whose solution it would be helpful to have an easy and inexpensive method for altering the order of the rows and columns of a matrix. One example of this kind of problem is the attempt to find a Spearman hierarchy in a correlation matrix. Another is cluster analysis or the selection of highly inter-related subgroups of tests for such factoring procedures as the grouping or multiple-group methods. Perhaps the most notable example is Guttman's scalogram analysis.† In fact, this paper might well have been entitled "A Cheap Scalogram Board."

In scalogram analysis the matrix to be rearranged is the score matrix, in which the rows represent the members of the experimental sample, while the column headings are the response categories for questionnaire items. In any row of this matrix, the only cells which are filled are those corresponding to the response categories which the individual involved has endorsed. These entries are 1's or X's or check marks, while all other cells are regarded as containing zeros. The task of scalogram analysis is first to reorder the rows of this matrix and then to shuffle the columns in such a way as to come out with the closest approximation to a parallelogram pattern of the non-zero entries. To accomplish this, Guttman has invented the scalogram board, a device consisting of a rack which holds a hundred narrow strips of wood, each strip representing a row of the score matrix. There are a hundred recesses drilled into each strip to represent one hundred cells in that row of the matrix. The response pattern for any person is indicated by placing buckshot or small ball bearings in the recesses corresponding to the response categories he checks. When this is done for every person, the rows of the score matrix can be reordered to suit the investigator. When the time comes to interchange columns, a second board, identical with the

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first, is placed upside down on top of the first board, with its wood strips at right angles to those of the first board. The two boards are then held together in that relationship and turned over, so that the balls in the first board fall into the corresponding recesses in the second board. The first board (which is now on top and completely empty of balls) is then removed, and the investigator can proceed to rearrange the columns of the score matrix, for the movable strips in the second board now represent those columns.

Perhaps the main disadvantage of the scalogram board is its prohibitive cost, which is greatly increased by the precision of manufacture that is necessary in order that all balls from the first board will fall freely into the second when the two are turned over. Great uniformity is thus required in the spacing of the recesses and in the widths of the strips. Cost estimates run into the hundreds of dollars. A second disadvantage that might be mentioned is the time required to place the balls in their proper positions in the board. Even the dropper which has been designed for this work* will not be nearly so fast as the placing of check marks in the proper cells of a data sheet. A third drawback of this method for rearranging matrices is that it is applicable to tables of qualitative data only. That is, the score matrix may show only presence or absence of an attribute (indicated by presence or absence of a ball in a recess) and cannot reflect quantitative differences such as are usually present in mental test score matrices and in correlation tables. This drawback is of no consequence for scalogram analysis itself, which deals exclusively with qualitative data, but it prevents the use of scalogram boards in reordering many other types of matrices. One possible way to overcome this defect would be to employ beads of different colors in place of the metal balls, but there would still be a limit to the number of differentiable colors that could be used. A fourth restriction is that the capacity of the board, in terms of the maximum number of columns and rows it can represent, is fixed once the board has been constructed.

Let us now take up the description of a simple procedure which will overcome the disadvantages of the scalogram board that have been mentioned, while at the same time introducing no serious new drawbacks of its own. The matrix to be rearranged is first recorded on an ordinary data sheet. To avoid certain difficulties later on, it may prove desirable to utilize only every other column and row in this recording process. The completed data sheet is taken to an ordinary paper cutter and is cut into strips in such a way that each strip is a row of the original matrix. If the narrow strips of the data paper tend to twist or curl either immediately or after considerable handling, the sheet can be fastened, before cutting, to a piece of stiff cardboard by strips of masking tape applied to its right and left edges. Great uniformity in the cutting process is unnecessary. The resulting strips can

*Ibid., p. 96.