THE ESTIMATION OF THE DISCRIMINAL DISPERSION IN THE METHOD OF SUCCESSIVE INTERVALS*

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A new algebraic formula is derived for estimation of the discriminal dispersion in the method of successive intervals. The legitimate use of the formula requires that as many normal deviates as possible be present in the matrix. For this reason, it is recommended that deviates corresponding to the interval (0.01, 0.99) of the cumulative proportions be used, instead of those corresponding to (0.05, 0.95), the interval used by Edwards and Thurstone. Computations on data published by Edwards and Thurstone showed that when adjustment was made for variability in dispersions calculated by the formula of this paper, a reduction of fifty per cent in mean absolute discrepancy was produced. Since the formula is easy to use and avoids the disadvantages of its predecessors, it should have fairly wide applicability in psychological research.

The method of successive intervals is perhaps the most practical way of obtaining rational scale values of stimuli along a unidimensional psychological continuum not simply correlated with any physical variable. The data may be provided by any procedure in which judges classify stimuli into a finite number of mutually exclusive and exhaustive classes which are ordered along some dimension.

When the number of stimuli is small, they may be ranked without ties, so that the number of classes equals the number of stimuli. When the number of stimuli is large, they may be either sorted into piles or rated on a rating scale. With either of these procedures, the number of classes may be considerably less than the number of stimuli. For adequate reliability, a large sample of judges is needed when any of these techniques of gathering data is used.

Although successive intervals was developed by L. L. Thurstone, its first published account was given in a paper by Saffir (8) in 1937. Recently, papers by Edwards (4) and Edwards and Thurstone (5) have presented a

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check on internal consistency which indirectly tests the applicability of the postulates to any particular set of data. This check now makes successive intervals a serious rival to the method of paired comparisons. The advantage of successive intervals over paired comparisons lies in its greater speed in collecting data. Empirical studies (4, 5) have shown that there is a linear relation between scale values obtained by these two methods.

In any stimulus scaling method developed in the Thurstone manner, there are at least two important kinds of parameters, represented respectively by $S_i$, the scale value of the $j$th stimulus, and $\sigma_i$, the corresponding discriminant dispersion. Although adequate computational techniques for estimating each $S_i$ by the method of successive intervals have been published (4, 5), those available for estimating $\sigma_i$ are subject to improvement.

The first technique, developed by Thurstone and presented by Saffir (8), does not base the computation of each $\sigma_i$ on all of the data. Also, it does not use a simple algebraic formula in the manner originated by Thurstone (9, 10) and further applied by Burros (2) and Burros and Gibson (3) for estimation of $\sigma_i$ in the method of paired comparisons. It is interesting to note, therefore, that in a recent paper on successive intervals, Edwards and Thurstone (5) did not use the technique presented by Saffir for estimating the dispersions. Instead these writers used one published by Attneave (1). A critical examination of Attneave's technique will be made later on in this paper. In the writer's opinion, it does not have a rigorous basis.

Perhaps the most rigorous approach to the problem is a least squares solution recently published by Gulliksen (6). Unfortunately, it is possible (although admittedly improbable) that negative estimates of the dispersions may be calculated by this technique. This sort of result could happen if the dispersions are exceedingly variable. A small positive dispersion could then be estimated as negative when his least squares solution is applied to the data. A related discussion of this problem of absurd results in paired comparisons is presented by Burros and Gibson (3, pp. 63–64).

Since the techniques published by Saffir (8) and Attneave (1) are questionable, and the one by Gulliksen (6) conceivably may give absurd results, a new formula may be of interest. This paper, therefore, presents the derivation of a simple formula for the estimation of $\sigma_i$ in the method of successive intervals, which is similar to those previously derived for paired comparisons (10, 2, 3). The use of the formula will then be illustrated by means of further analysis of data presented by Edwards and Thurstone (5).

**Definition of Symbols**

$R$ = postulated unidimensional psychological continuum with finite range arbitrarily divided into $N$ class intervals corresponding to the steps on an $N$-point rating scale;