A STATISTICAL MODEL FOR RELATIONAL ANALYSIS*

R. Duncan Luce† and Josiah Macy, Jr.‡
Massachusetts Institute of Technology

and

Renato Tagiuri
Harvard University

The diadic relationships existing in a group can be defined in terms of the members' choices, rejections, and their perceptions of being chosen and rejected. The number of possible distinct diads is 45. Formulas are given for computing the expected frequency and variance of the different diadic forms expected, when certain random factors are taken into account. These values must be known if the operation of factors other than the specified random ones is to be studied. Values obtained from two models with different assumptions are compared with empirical values. A simplified treatment is possible for groups with ten or more members.

The student of interpersonal processes often needs to describe and classify in some useful form the relationships between individuals. One such classification is given by relational analysis (2), a method developed in conjunction with a series of studies in interpersonal perception. In this classification the relationship between two persons is described in terms of the feeling each has for the other, and the perception each has of the other's feeling. More specifically, each member of a well-acquainted group is asked to select those he likes most and those he likes least, and also to guess who likes him most and least. This procedure yields a simple but useful description of the relationship existing between each of the $N(N - 1)/2$ pairs in the group.

Since each subject $S_i$ can choose, reject, or omit any other subject $S_j$, and can feel chosen, rejected, or omitted by him, nine arrangements are possible of $S_i$'s feelings and perceptions regarding $S_j$. We shall define a diad between $S_i$ and $S_j$ as any one of the nine possible arrangements of selections of $S_i$, combined with any one of the nine possible arrangements of selections of $S_j$, without regard to order. The number of possible distinct diads is 45.

If $S_i$'s feeling toward $S_j$ be denoted by 0 for like, 1 for omission and 2 for dislike and, if $S_i$'s predictions of $S_j$'s feeling toward him be denoted by

---

*The present problem emerged from research undertaken as part of a project in interpersonal perception being carried out at the Laboratory of Social Relations at Harvard with the financial aid of the Office of Naval Research (Task Order N5or1—07646).
†Now at Columbia University.
‡Now at Johns Hopkins University.
0 for like, 1 for omission and 2 for dislike, then we can represent some of the 45 possible relationships as follows:

\[
\begin{array}{cccc}
S_i & S_i & S_i & S_i \\
(11) & (11) & (00) & (00) \\
(01) & (11) & (00) & (22) \\
(00) & (01) & (22) & (22) \\
\end{array}
\]

Legend: the first digit in each bracket corresponds to the feeling, the second to the perception.

Some of the possible diads are well integrated, positive, and realistic; others involve contrary feelings and mistaken perceptions; still others indicate a well-developed negative, and recognized, mutual orientation.

Psychologically important features of a group can be described in terms of the frequency of occurrence of the various diads. It is apparent, however, that given the number of choices, omissions, and rejections, and the number of perceptions of choice, omission, and rejection made by each member of a given group, each diad may be expected to occur a certain number of times by chance alone. To interpret observed data we must know something of these chance distributions, so that we will not attempt to give a psychological interpretation to data which can be explained by the operation of chance alone. When we know which specific diads occur from group to group with greater or less than chance frequency, then we can formulate hypotheses about the possible non-chance factors at work. For these reasons it is important to be able to state the expected frequency of occurrence and the variance of each diad type in a group of given size for an assumed chance model. In previous work (3) estimates of these quantities were obtained by constructing a Monte Carlo robot “group,” which was set to match the real group man by man in the number of choices, omissions, and rejections made and in their respective perceptions. This is clearly an unsatisfactory and inefficient method if it can be replaced by a simple mathematical formula.

The purpose of this paper is to present a model in terms of which we can estimate the expected chance frequency and variance of the various diads. It should be borne in mind that the distribution of such frequencies is, probably, often more Poisson than normal.

I. The Model

Several possible “chance” models are conceivable, depending on what we choose to regard as chance. The first one we shall examine corresponds to the case in which the members of a group are regarded as automata, randomly allocating their selections according to fixed probabilities of choosing, rejecting, or omitting every member of the group. Three other