A METHOD OF SCALOGRAM ANALYSIS USING SUMMARY STATISTICS*

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A method of Guttman scalogram analysis is presented that does not involve sorting and rearranging the entries in the item response matrix. The method requires dichotomous items. Formulas are presented for estimating the reproducibility of the scale and estimating the expected value of the chance reproducibility. An index of consistency is suggested for evaluating the reproducibility. An illustrative example is presented in detail. The logical basis of the method is discussed. Finally, several methods are suggested for dealing with non-dichotomous items.

Guttman's scaling method, known as scalogram analysis (4), has become popular among social scientists. However, current techniques for scalogram analysis are cumbersome. They all deal directly with the raw data in the form of an item response matrix that has a row for each respondent and a column for each item response category. An entry in the matrix indicates whether a particular respondent gave a particular item response. Various procedures have been described for rearranging the rows and columns of the item response matrix, as well as for combining response categories, so that a response "parallelogram" is achieved with few deviations. Suchman (12) described the scalogram board procedure in which the response matrix is represented by buckshot placed in small indentations in a set of removable slats. The sorting is accomplished by interchanging these slats in their frame. Methods for tabulating the response matrix on IBM equipment have been described by Noland (10), Ford (2), and Kahn and Bodice (6). Paper and pencil methods have been described by Guttman (3) and Marder (8).

These techniques are not automatic, but require keen judgment concerning the kind of sorting likely to pay off. Furthermore, the techniques are cumbersome since each attempts to evaluate the complete raw data matrix without the aid of summary statistics. For large numbers of respondents, the task is overwhelming. Moreover, it is difficult to deal with more than 10-20 items with these procedures. [A method, called H-technique, for combining items before making the scalogram analysis has been reported by Stouffer, Borgatta, Hays, and Henry (11)].

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The purpose of this paper is to present a relatively simple method of scalogram analysis in which summary statistics are used to compute a close approximation to the scale \( \text{rep} \). (In this paper “rep” is used for “reproducibility.”) In this method, which requires dichotomous items, there is no limitation on the number of respondents; its application to large numbers of items is relatively easy. The method is particularly well-suited to punched-card techniques of processing data, since one must merely count the number of respondents who gave the positive response to each item, and the number of respondents giving certain specified combinations of responses. Obtaining these summary statistics is a simple, routine, completely objective matter.

In a sense, the method proposed here removes scalogram analysis from the list of subjective, slightly mystical techniques available only to experienced practitioners and places it on the list of objective methods available to any statistical clerk. The method also substantially reduces the time required for analysis. It gains these advantages at the expense of providing only an approximation to the “true” \( \text{rep} \). Certain high-order scale errors are ignored. However the approximation appears to be a very close one.

**The Method**

All items must be dichotomous. In the mathematical notation we will let \( k \) be the number of items, \( N \) be the number of respondents, \( i \) be a subscript referring to item \( i \) (where the items are in any arbitrary order), and \( g \) be a subscript referring to item \( g \) in rank order.

**Step 1.** Designate the positive response to each item by referring to the item content. The positive response designations should be consistent with the investigator’s hypothesis concerning the dimension being scaled.

**Step 2.** For each item tabulate \( n_i \), the number of respondents who gave the positive response to the item.

**Step 3.** Arrange the items in rank order according to their popularities, \( (n_i/N) \), with the least popular item getting rank \( k \), and the most popular item getting rank 1. If there are any ties, adopt an arbitrary order.

**Step 4.** Tabulate \( n_{g+1,g} \) for \( g = 1, 2, \ldots, k - 1 \). This is the number of respondents who gave the positive response to item \( g + 1 \) and the negative response to item \( g \). If it is easier to tabulate \( n_{g+1,g} \) or \( n_{g+1,g} \), then the following identities can be used:

\[
 n_{ij} = n_i - n_{ij} ; \\
 n_{ij} = n_{ij} + n_i - n_i .
\]

**Step 5.** Use either of the following two methods for estimating the \( \text{rep} \).

1. Tabulate \( n_{g+2,g+1,g-1} \) for \( g = 2, 3, \ldots, k - 2 \). This is the number of respondents who gave the positive response to item \( g + 2 \), and the positive response to item \( g + 1 \), and the negative response to item \( g \), and the negative