THE VARIMAX CRITERION FOR ANALYTIC ROTATION IN FACTOR ANALYSIS

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An analytic criterion for rotation is defined. The scientific advantage of analytic criteria over subjective (graphical) rotational procedures is discussed. Carroll's criterion and the quartimax criterion are briefly reviewed; the varimax criterion is outlined in detail and contrasted both logically and numerically with the quartimax criterion. It is shown that the normal varimax solution probably coincides closely to the application of the principle of simple structure. However, it is proposed that the ultimate criterion of a rotational procedure is factorial invariance, not simple structure—although the two notions appear to be highly related. The normal varimax criterion is shown to be a two-dimensional generalization of the classic Spearman case, i.e., it shows perfect factorial invariance for two pure clusters. An example is given of the invariance of a normal varimax solution for more than two factors. The oblique normal varimax criterion is stated. A computational outline for the orthogonal normal varimax is appended.

In factor analysis, an analytic criterion for rotation is defined as one that imposes mathematical conditions beyond the fundamental factor theorem, such that a factor matrix is uniquely determined. Historically, the first such criterion was Thurstone's treatment of the principal axes problem [10]: from any arbitrary factor matrix he suggested rotating under the criterion that each factor successively accounts for the maximum variance. But principal axes have seldom been accepted as psychologically very interesting ([9], p. 139). The rotation problem for psychologically meaningful factors is usually handled judgmentally. Scientifically, however, this procedure is not very satisfactory: the ad hoc quality of subjective rotation makes uniquely determined factors impossible; only factors that are subject to the uncertainties and controversies besetting any a posteriori reasoning can be defined. In contrast, an analytic criterion for rotation would allow factor analysis to become a straightforward methodology stripped of its subjectivity and a proper tool for scientific inquiry.

The Quartimax Criterion

The first analytic criterion for determining psychologically interpretable factors was presented in 1953 by Carroll [1]. In an attempt to provide a

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mathematical explication of Thurstone's simple structure, he suggested that for a given factor matrix,

\[ f = \sum_{s < t} \sum_{j} a_{js}^2 a_{jt}^2 \]

should be a minimum, where \( j = 1, 2, \ldots, n \) are tests, \( s, t = 1, 2, \ldots, r \) are factors, and \( a_{js} \) is the factor loading of the \( j \)th test on the \( s \)th factor.

It appears that Carroll was motivated in writing (1) primarily by a close inspection of Thurstone's five formal rules for simple structure ([12], p. 335), particularly the requirement that a large loading for one factor be opposite a small loading for another factor.

In his original paper, Carroll provided two numerical examples of the application of his method. Without the restriction of orthogonality, these illustrations gave somewhat equivocal results—while the application of (1) appears to bring one close to the desired simple structure, the criterion has an obvious bias in being too strongly influenced by factorially complex tests.

In the light of later developments, Carroll’s criterion should probably be relegated to the limbo of “near misses”; however, this does not detract from the fact that it was the first attempt to break away from an inflexible devotion to Thurstone’s ambiguous, arbitrary, and mathematically unmanageable qualitative rules for his intuitively compelling notion of simple structure.

Almost simultaneously with Carroll’s development, Neuhaus and Wrigley [7], Saunders [8], and Ferguson [2] proposed what is usually called the quartimax method for orthogonal simple structure. Neuhaus and Wrigley suggest that a most easily interpretable factor matrix, in the simple structure sense, may be found when the variance of all \( nr \) squared loadings of the factor matrix is a maximum, i.e.,

\[ q_1 = [nr \sum_i \sum_s (a_{is}^2)^2 - (\sum_i \sum_s a_{is}^2)^2] / nr^2 = \text{maximum}. \]

Saunders’ approach requires that the kurtosis (fourth moment over second moment squared) of all loadings and their reflections be a maximum,

\[ q_2 = nr \sum_i \sum_s a_{is}^4 / (\sum_i \sum_s a_{is}^2)^2 = \text{maximum}. \]

While Ferguson, basing his rationale on certain parallels with information theory, calls simply for

\[ q_3 = \sum_i \sum_s a_{is}^4 = \text{maximum}. \]

All these investigators are concerned with attaining a factor matrix with a maximum tendency to have both small and large loadings. While less obviously related to Thurstone’s rules than Carroll’s criterion, the emphasis on small loadings coincides with Thurstone’s requirements of