STRUCTURAL CENTRALITY IN
COMMUNICATIONS NETWORKS

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This paper examines the concept of centrality with respect to small-group communication experiments. An index of centrality is presented which is based on the incidence matrix of actual communications rather than on the deviation matrix of possible communications, as in the Bavelas Index of Centrality. The index takes the value of zero for the homogeneous all-channel graph and the value of unity for the homogeneous wheel graph. The index can be computed for individuals as well as groups. Three examples are computed.

Small-group communications network experiments use graphs like the wheel, circle, star, chain, and all-channel to represent the flow of messages or interactions. This paper examines the concept of structural centrality and presents a new measure based on the actual number of messages sent to the participants. A communications network is considered structurally centralized to the degree that the network approaches that of a wheel network and decentralized to the degree that the graph is that of an all-channel.

The network or graph $G$ consists of $n$-participants $G_1, G_2, \ldots, G_n$ and the communication arcs or channels between the participants. Writers like Bavelas [1, 2], Leavitt [7], Flament [5] associate with every communications network a matrix of ones and zeros. Such a matrix is called a sociomatrix. Unity is placed in the $ij$th entry if it is possible for $G_i$ to send a communication to $G_j$; the $ij$th element is zero if this interaction is not possible. This particular matrix formulation has two basic weaknesses.

(i) Often some of the possible channels are not used.
(ii) Those channels that are used are not always used in the same amount.

A sociomatrix of a small group is not an accurate representation of the actual communication pattern unless all possible channels are used equally. The participants may chose not to use all of the possible communication channels

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or they may use them unequally. For example, $G_i$ may send seven messages to $G_j$, only two to $G_k$, and none to $G_m$.

A more realistic approach is to construct incidence matrices. (The definition given in this paper differs from that of Ford and Fulkerson [6] or Berge [3] where an incidence matrix is defined as a matrix $S = [s_{ij}]$ where $s_{ij}$ is 1 if the arc is from $G_i$ to $G_j$, -1 of from $G_i$ to $G_j$, and 0 if there is no arc between $G_i$ and $G_j$). An incidence matrix $Q = (q_{ij})$ is a (square, hollow, Frobenius) matrix whose elements are defined by

\[
q_{ij} = \begin{cases} 
\frac{\text{number of communications sent by } G_i \text{ to } G_j}{\text{total number of messages sent by } G}, & \text{for } i \neq j, \\
0, & \text{if } i = j.
\end{cases}
\]

The definition of $Q$ implies that the sum of the $q_{ij}$ is unity. In general, $q_{ij} \neq q_{ii}$. This definition of $Q = [q_{ij}]$ gives dimensionless entries (by virtue of being dimensionless, incidence matrices of groups of the same number of participants but sending different numbers of messages can be compared) that vary in size with the intensity of communications between the $n(n - 1)$ possible arcs. The two major weaknesses of the sociomatrix are not present in the incidence matrix.

One possible weakness of the incidence matrix is that it may be harder to conceptualize the structure of the network. For instance from the sociomatrix it is possible to construct a deviation matrix $D = [d_{ij}]$, where the $ij$th entry is the minimum number of arcs on a path from $G_i$ to $G_j$. When the arcs are characterized by $q_{ij}$ rather than 1's and 0's, the interpretation of the structure appears more complicated. (The deviation matrix can be used, by itself, to construct the main topological features of the network $G$. But in terms of describing communication patterns, it does not seem useful except in the case where all the possible channels are used equally and messages are relayed without restatement and/or interpretation). However this possible weakness would appear to be more than compensated by the ability to specify a more detailed (and realistic) measure of centrality.

In the traditional approach, the concept of a deviation matrix, leads directly to the Bavelas Index of Centrality [1]. The latter is defined on the deviation matrix $D = [d_{ij}]$ by summing the rows to get

\[
d_i = \sum_{j=1}^{n} d_{ij}
\]

and summing the $d_i$ to get $D$. The index of centrality is given by

\[
\sum_{i=1}^{n} \frac{D}{d_i}.
\]

This measure is not very sensitive to large shifts in the sociomatrix. For example, the centrality index of a wheel is 26.3 and 25.0 for the circle and all-channel.