AN I.B.M. TECHNIQUE FOR THE COMPUTATION OF $\sum x^2$ AND $\sum xy$*

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Given I.B.M. cards punched with scores (or any numbers) but not their squares—a method is presented of tabulating them (on the No. 405 alphameric I.B.M. tabulator) so as to obtain the sum of squares. The technique is also adaptable to summation of cross-products. The principle is an extension of the Mendenhall-Warren-Hollerith technique of vertical progressive digiting, without the necessity of manual addition or summary-punching, and is designed for machines not equipped with the “card cycle total transfer” device or “progressive total” device. Use is made of “counter rolling.” Efficient use of machine capacity is made only when intercorrelations between no more than two variables are required in addition to sums of squares. A résumé of some techniques now commonly employed is included.

To obtain a sum of squares, the following are some of the I.B.M. methods now in common use, when detail cards contain scores (or any numbers), but not their squares:

A. Selecting one master squares card for every detail card. This requires a collator and a previously prepared master file (which may, however, also contain other data—such as higher powers). Matched masters are subsequently tabulated, and then re-merged with the file (8).

B. Interspersed master gang-punching. This also requires prepared squares deck, use of gang-punch machine, and subsequent tabulation of detail cards.

C. Use of automatic multiplying punch. Summary-products counter will contain sum of squares; or, squares can be punched into details and these tabulated (3).

D. Horizontal digiting, requiring at least one digit selector, and three counter-groups per variable. This method also requires multiplication and addition of totals (5).

E. Mendenhall-Warren-Hollerith correlation method: printing of

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vertically progressively digited totals—from highest to lowest score. This requires "progressive total" device, manual addition after tabulation and allowance for gaps in the distribution (1, 2, 9).

F. Summary-punching vertically progressively digited totals. This requires digit cards (for possible gaps in the distribution), summary-punch in conjunction with tabulator (equipped with "progressive total" device), and subsequent tabulation of summary cards.

G. Transfer of vertically progressively digited totals to another counter (which will finally contain the sum of squares) by use of "card cycle total transfer" device. Digit cards are required, but no "progressive total" device (6).

Sorting is required in all cases, except (C) and (D).

The method here described is similar to (G), but designed for machines not equipped with "card cycle total transfer" device. It is based on the same mathematical principle as (E), (F), and (G) (see 4). A detailed explanation of the wiring, with explanatory notes, is given in an Appendix.

**Necessary Equipment**

a) Sorter; b) alphameric tabulator No. 405 (I.B.M.), equipped with at least 2 class selectors, and a number of independent "X"-distributors equal to the number of digits expected in the sum of scores (5 were allowed for in the appended wiring directions). "D" pick-up hubs must be operative; but neither "progressive total" device nor digit selector is needed. c) 2 digit cards for every possible score from the highest actual down to 1 (there should be none for zero scores), with scores punched in the same fields used in detail cards. Digit cards should contain an identifying punch, and be sorted behind details, all in order from highest to lowest score.

For example, assume the following scores:

<table>
<thead>
<tr>
<th>STUDENT NO:</th>
<th>I</th>
<th>II</th>
<th>III</th>
<th>IV</th>
<th>V</th>
<th>VI</th>
<th>VII</th>
<th>VIII</th>
<th>IX</th>
<th>X</th>
<th>XI</th>
</tr>
</thead>
<tbody>
<tr>
<td>SCORE:</td>
<td>12</td>
<td>4</td>
<td>12</td>
<td>9</td>
<td>11</td>
<td>9</td>
<td>11</td>
<td>8</td>
<td>11</td>
<td>11</td>
<td>0</td>
</tr>
</tbody>
</table>

*The order of cards will be:*

Detail card with 12 in columns 7-8

```
12
```

Digit

```
12
```

Detail

```
11
```

```
11
```

and "X" in column 80

```
"80
```

"" "" "" 7-8

"" "" "" 7-8

"" "" "" 7-8

"" "" "" 7-8