RELATIONS BETWEEN MULTIDIMENSIONAL SCALING AND THREE-MODE FACTOR ANALYSIS*

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A combination is achieved of two lines of psychometric interest: a) multidimensional scaling and b) factor analysis. This is accomplished with the use of three-mode factor analysis of scalar product matrices, one for each subject. Two of the modes are the groups of objects scaled and the third mode is the sample of subjects. Results are an object space, a person space, and a system for changing weights given to dimensions and of angles between dimensions in the object space for individuals located at different places in the person space. The development is illustrated with data from an adjective similarity study.

One line of recent development in quantitative psychology involves creation of models which incorporate description of individual behavior with description of the variety of individuals as to this behavior. Multidimensional scaling of individual responses in comparisons among objects in a group of objects is a particular example. A separate scaling experiment could be conducted for each individual in a sample of subjects so as to obtain measures of dissimilarity between objects or pairs of objects. Such measures of dissimilarity frequently are taken as distances between points for the objects in a space which represents the responses of the subject to the objects. Tucker and Messick [1963] presented a model for investigating the variety of such spaces for a sample of individuals. An example of the application of this model to color vision data was given by Helm and Tucker [1962]. Tucker and Messick emphasized the description of the subjects by the establishment of a person space. The multidimensional scaling space implied for selected "idealized individuals" could be determined by subsequent analysis and used as an aid in interpretation of the person space.

In contrast to the Tucker-Messick [1963] model which did not specify the nature of differences between multidimensional scaling spaces for various individuals, a model presented by Horan [1969] involved a common multidimensional space for all individuals and limited differences between indivi-

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duals to differential weights applied to dimensions of the common space. Horan did not develop a procedure for explicit description of individual persons. Carroll and Chang [1970] utilized a model similar to that of Horan and achieved a unique weight matrix, thus providing descriptions of individuals.

A more general view of the variety of psychological spaces utilized by the variety of persons should allow, however, not only differential weightings of the dimensions of a common space but also differential relations among these dimensions. In a space representing the relations among attitude items, for example, there may be one psychological dimension representing attitude toward communism and another psychological dimension representing attitude toward the church. For some persons these dimensions may be independent. To some other persons these dimensions may be related to varying degrees with an extreme case of equating pro-communism to anti-church and anti-communism to pro-church, thus combining the two dimensions into one. These differential relations among psychological dimensions may be represented in the multidimensional scaling model by differential angles between the dimensions. Such changing angles between persons probably have great psychological importance in our understanding of the variety of behavior of the variety of individuals. Some pathological syndromes may be characterized by bizzare distortions of a psychological space involving combinations of dimensions. At more normal levels, failures of individuals to understand each other may originate from differential obliqueness of their psychological spaces, the differential obliqueness leading to differing assumptions as to relations between objects, tendencies for the objects to go together or to be in opposition. These differing psychological spaces may be one of the sources of controversy and argument. They may affect the interpretation of common observations and the nature of decisions made by different persons. Differential skewing of a psychological space by various persons probably differentially colors their psychological being.

Tucker's [1966] recent development of three-mode factor analysis yields a model which involves both a person space and a common object space for which weights given to the dimensions and angles between the dimensions are functions of the person parameters in the person space. This model applies not only to multidimensional scaling but also to many other situations. It may be used whenever each object in a group of objects is represented as a vector in an object space, each vector emanating from a common origin and extending to a terminal point characterizing the object. Input data to analysis by the model are the scalar products of the pairs of vectors for each subject. Such scalar products may be obtained by some one of a variety of procedures, including direct ratings as in the data used in an example to be presented. For multidimensional scaling when measures of interpoint distances in a Euclidean space are available, these distances may be converted to