JACKKNIFING DISATTENUATED CORRELATIONS

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The utility of the jackknife for constructing confidence intervals and testing hypotheses about the disattenuated correlation is evaluated for small samples. Computer simulations were used to generate the empirical sampling distributions of jackknife statistics for two sample sizes (30, 60), five values of the disattenuated correlation coefficient (1.00, .90, .80, .50, .00) and three pairs of reliabilities (.90, .80; .80, .80; .90, .50). The theoretical and cumulative proportions of jackknife statistics were compared at selected points in the appropriate t-distributions. The results obtained support the claim that the jackknife can be used to construct sensible confidence intervals. However, the jackknife possesses limited utility for testing hypotheses about the disattenuated correlation coefficient.

Despite authoritative support for the use of correlations corrected for attenuation [Gulliksen, 1950; Block, 1963; Lord, 1957, 1970; Lord and Novick, 1968; Cochran, 1970; Cronbach, 1971], their actual use has been rather limited. This may be explained in part by inadequate knowledge about the sampling distribution of the sample estimator and the resulting lack of general inferential procedures. Unlike the uncorrected coefficient, the theoretical sampling distribution for the disattenuated correlation coefficient has not been derived. The complexity of the estimator leads to intractable mathematics, making an exact analytical solution exceedingly difficult, if not impossible.

In the absence of adequate sampling theory, several inferential techniques have been proposed. These procedures have proved to be either cumbersome, approximate, or restricted to specific hypothesis and/or to large samples. The tests developed by Lord [1957; 1971], McNemar [1958], and Forsyth and Feldt [1970] are restricted to the hypothesis that the disattenuated correlation is equal to one. Lord's test [1957] is based on the large sample $\chi^2$ approximation to the likelihood ratio statistic. McNemar's analysis of variance approach, although appropriate for any sample size, assumes that the tests are equally reliable. Forsyth and Feldt's modifications of McNemar's

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test statistic for unequal reliabilities provides only an approximate solution and should not be used when the reliabilities are markedly different.

Forsyth and Feldt [1969], Jöreskog [1971] and Kristoff [1972] proposed procedures for testing hypotheses and constructing confidence intervals about the disattenuated correlation when the relation is less than perfect. The normal curve method suggested by Forsyth and Feldt is a large sample procedure and requires the use of a complex formula for obtaining approximate values for the standard error. Jöreskog's procedure, which uses maximum likelihood methods, is also a large sample test. Kristoff's technique, designed to test whether there is any linear relation, uses a correlational approach which tends to yield conservative results.

The jackknife [Tukey, 1958] is a general algorithm intended to provide good approximate confidence intervals in situations where known procedures do not exist, are restricted to large samples, or are difficult to apply. Miller [1974] has provided a comprehensive review of the research and several applications built upon Tukey's initial idea. In addition the jackknife has been profitably applied to factor loadings [Pennel, 1972] and in multiple matrix sampling [Shoemaker, 1973]. However, as Miller pointed out the jackknife is not foolproof. Earlier, Miller [1964] and Mosteller and Tukey [1968] warned of difficulties where the sampling distribution of the estimator is asymmetrical or where the values of the parameter being estimated are restricted to an interval or half-line. For example, Wainer and Thissen [1975] found that although the jackknife provided a realistic estimate of the standard error of the uncorrected correlation coefficient for small samples, it failed to provide a reasonable point estimate when there were sharp deviations from normality.

Arveson [1969] provided a large sample rationale for the use of the jackknife with correlations corrected for attenuation. Previous work by Forsyth and Feldt [1969] showed that, for small samples, the sampling distribution of the disattenuated correlation coefficient is not symmetrical. In light of this finding and in view of the warnings of Miller, Mosteller, and Tukey, the present study was designed to evaluate empirically the generality of the use of the jackknife with the disattenuated correlation coefficient for small samples, using Monte Carlo techniques.

The Jackknife

Let $\theta$ be the unknown parameter and let $\{X_1, X_2, \ldots, X_n\}$ be a random sample of $n$ independent, identically distributed observations. The jackknife requires that these observations be divided into $k$ groups of size $n'$ ($n' > 1$) such that $n = n'k$. Let $\hat{\theta}$ be an estimator of the parameter $\theta$ based on all $n$ observations and let $\hat{\theta}_{(i)}$, $i = 1, \ldots, k$, denote the corresponding estimator based on the $(n - n')$ observations in the subsample obtained by omitting the $i$th group. Pseudo-values,