A RATING FORMULATION FOR ORDERED RESPONSE CATEGORIES

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A rating response mechanism for ordered categories, which is related to the traditional threshold formulation but distinctively different from it, is formulated. In addition to the subject and item parameters two other sets of parameters, which can be interpreted in terms of thresholds on a latent continuum and discriminations at the thresholds, are obtained. These parameters are identified with the category coefficients and the scoring function of the Rasch model for polychotomous responses in which the latent trait is assumed unidimensional. In the case where the threshold discriminations are equal, the scoring of successive categories by the familiar assignment of successive integers is justified. In the case where distances between thresholds are also equal, a simple pattern of category coefficients is shown to follow.

Key words: logistic latent trait, Rasch model, thresholds, ordered categories.

1. Introduction

An interesting and perhaps puzzling feature of the Rasch model for polychotomous responses in which the latent trait is presumed unidimensional is that the category coefficient and scoring function parameters of the model have not been interpreted fully with respect to any response mechanism. The main purpose of this paper is to specify a rating mechanism which leads to such an interpretation. Andersen [1977] has derived certain statistical properties of the model and in a sense therefore, this paper is complementary to Andersen’s.

We commence our development by tracing briefly the circumstances which have created this unusual case of uninterpreted parameters. We do this in Section 2 within the context of dichotomous response models. In Section 3 we outline generalizations of these models to polychotomous cases. In Section 4, and from the first stage of the traditional formulation, we propose a response mechanism which leads to the model identifiable with the Rasch model. In Section 5 we consider some implications of the obtained model and we summarize these in Section 6.

2. Dichotomous Response Models

In this section we highlight certain features of dichotomous response models necessary to place the subsequent development into context.

In latent trait theory for dichotomous test items, two distinct types of formulations seem to have evolved. The first, exemplified by Lord [1952] and Birnbaum [1968], has its origins in specifying plausible subject and item parameters together with a plausible probability distribution for a correct response. The second, exemplified by Rasch [1961, Note 1] and Andersen [1977], has its origins in specifying certain conditions for the estimation of subject and item parameters and then deducing the form of the possible probability distribution.

This work was conducted in part in the first half of 1977 while the author was on study leave at the Danish Institute for Educational Research. The Institute provided required research facilities while The University of Western Australia provided financial support.

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In the former, a subject ability parameter $\beta$ and two item parameters, the difficulty $\delta$ and discrimination $\alpha$, are postulated. These parameters, found in traditional test theory and pertaining to the ubiquitous location and dispersion properties of distributions, are clearly plausible. Then if $X$ is a random variable which takes the value $x = 1$ for a correct response and 0 otherwise, the probability distribution proposed by Lord is

$$ p(X = 1 \mid \beta, \delta, \alpha) = \Phi(\alpha(\beta - \delta)) $$

where $\Phi$ is the cumulative normal distribution with mean zero and variance unity. The distribution suggested by Birnbaum is similar taking the form

$$ p(X = 1 \mid \beta, \delta, \alpha) = \Psi(\alpha(\beta - \delta)) $$

where $\Psi$ is the logistic function $\Psi(y) = \exp(y)/(1 + \exp(y))$.

The basic requirement specified by Rasch and Andersen is that the models have sufficient statistics for any subject and item parameters. For dichotomous responses, the Rasch model can take the form (2) except that it has no unknown discrimination parameter, that is,

$$ p(X = 1 \mid \beta, \delta) = \Psi(\beta - \delta). $$

From experience with the first type of formulation, the parameters $\beta$ and $\delta$ are then readily identified as the subject ability and item difficulty respectively. In fact, it is customary to consider the Rasch model as a special case of the Birnbaum model in which the common discrimination parameter is routinely absorbed into the other parameters as an arbitrary choice of unit. However, we must stress that Rasch does not arrive at his model by specializing the Birnbaum model but that he derives it from his requirement of sufficient statistics.

Partly because the models look so similar and partly because of the constant interplay between data, models, and the study of numerical methods, the distinctions between the two approaches have been blurred. However, they have different advantages and disadvantages which manifest themselves in correspondingly different ways.

An advantage in the first approach is that if one is not satisfied with the parameterization of a model, one can suggest new or restructured parameters in a way directed primarily by the response mechanism. Examples of this procedure are the addition of a guessing parameter to the item [Birnbaum, 1968], a guessing parameter to the subject [Waller, Note 4] and a discrimination parameter to the subject [Lumsden, 1977]. A disadvantage of this approach is that it may not be possible to estimate the parameters so introduced.

The advantage of the second approach, in which one is confined to a certain class of models, is that one knows in advance certain properties of the derived distributions. However, a possible disadvantage is that one may obtain a model which cannot be readily connected to any response mechanism generating the data. And this is the situation which has been reached with the Rasch model for polychotomous responses in which the item and subject parameters are assumed unidimensional.

### 3. Unidimensional Models for Polychotomous Responses

In this section we consider both the traditional and Rasch polychotomous response models for unidimensional traits. We follow the pattern of the previous section and once again only highlight certain features which will direct us to the required response formulation. To facilitate its development we note the following four points in advance.

First, we will find it convenient to shift from achievement testing situations to attitude testing for our illustrations. Thus instead of the dichotomous response to an item being correct or incorrect it becomes say agree or disagree. Then $\beta$ and $\delta$ are respectively interpreted as a subject's attitude and an item's inherent intensity or affective value.