FACTOR SIMPLICITY INDEX AND TRANSFORMATIONS

P. M. BENTLER
UNIVERSITY OF CALIFORNIA AT LOS ANGELES

A scale-invariant index of factorial simplicity is proposed as a summary statistic for principal components and factor analysis. The index ranges from zero to one, and attains its maximum when all variables are simple rather than factorially complex. A factor scale-free oblique transformation method is developed to maximize the index. In addition, a new orthogonal rotation procedure is developed. These factor transformation methods are implemented using rapidly convergent computer programs. Observed results indicate that the procedures produce meaningfully simple factor pattern solutions.

Key words: multivariate analysis, orthogonal rotation, oblique transformation.

The problem of assessing the degree of factorial simplicity in a given factor pattern matrix was recently considered in an important paper by Kaiser [1974], who showed that the quartimax criterion could be made relevant to assessing the tendency towards unifactoriality. Kaiser proposed an index varying between zero and one, with greater values of the index representing simpler and more interpretable solutions. The simplest possible solution would have each variable being generated by only a single common factor. The importance of the index lies in the possibility of comparing a variety of factor patterns, including those based on different content areas and numbers of variables and factors. Unfortunately, this index is sensitive to the scale of the factors, which is quite arbitrary, and so the conclusion one reaches about the simplicity of a given pattern matrix depends on an ad hoc convention in addition to intrinsic features of the data. Furthermore, Kaiser’s index is primarily a descriptive statistic for any given factor pattern. No provision has yet been made whereby the researcher can find that factor pattern for a given set of data that maximizes simplicity as defined by the index. An ideal index of factorial simplicity would not have these problems.

An understanding of the importance of factor scale freeness can be gained...
by examining the factor-analytic random vector model, which can be written as

\[ x = \mu + B\xi + \zeta, \]

where \( x, \mu, \) and \( \zeta \) are \( n \times 1 \) vectors, \( B \) is an \( n \times m \) matrix of common-factor "loadings," and \( \xi \) is an \( m \times 1 \) vector of common factor "scores." The mean vector is \( \mu \) and, on the typical assumptions of mutual orthogonality of common and unique factors \( \xi \) and \( \zeta \), as well as the mutual independence of unique scores, the covariance structure can be represented as

\[ \Sigma = B\theta B' + \Psi^2, \]

where \( \theta \) represents the covariance matrix of the \( \xi \) and \( \Psi^2 \) is the diagonal matrix of unique variances. Since \( \theta \) is a covariance matrix, it is appropriate to consider the corresponding correlation matrix given by \( \Phi = \Delta^{-1}\theta\Delta^{-1} \), where \( \Delta \) is a diagonal matrix. Thus (2) can be equivalently written as

\[ \Sigma = B\Phi B' + \Psi^2. \]

Using Harman's [1967] notation for the common factors, we can obtain the factor pattern matrix \( P = B\Delta \) and write the model as

\[ \Sigma = P\Phi P' + \Psi^2. \]

Except for the interpretive convenience associated with writing the factor model as (4), there is no particular reason to choose among the formally equivalent forms (2) and (4). Consequently, concepts of simple structure suggesting that a primary factor pattern matrix should possess some simple form must apply with equal force to (2) and (4). In other words, simple structure criteria should be stated such that they are independent of the choice of any arbitrary scaling diagonal \( \Delta \). This conclusion holds for the factor model as well as the principal components model with \( \Psi^2 = 0 \). Just as Kaiser's index was not developed to be invariant with respect to the scale of the factors, current analytical oblique simple structure criteria were not developed or rationalized in terms of scale-freeness. Nonetheless, it might be noted that among the well-known methods [Harman, 1967; Mulaik, 1972], a few can be considered to be scale-free [e.g., binormamin, Kaiser & Dickman, Note 2; oblimax (as implemented), Saunders, 1961; independent cluster, Harris & Kaiser, 1964]. In contrast, such methods as the classes of indirect oblimin [Carroll, Note 1] and direct oblimin [Jennrich & Sampson, 1966] solutions are scale-specific. Applying criteria such as oblimin to differing pattern matrices that are identical except for column scale (e.g., \( B \) vs. \( P \)) would consequently yield different function values for these matrices, implying that the different matrices do not have the same simple structure. An ideal simple structure criterion would not have this problem.

This paper proposes an index of factorial simplicity that is both invariant with respect to the scale of the factors and capable of being maximized, thus alleviating the two problems associated with Kaiser's index. The function that