A GENERAL LATENT TRAIT MODEL FOR RESPONSE PROCESSES

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The purpose of the current paper is to propose a general multicomponent latent trait model (GLTM) for response processes. The proposed model combines the linear logistic latent trait (LLTM) with the multicomponent latent trait model (MLTM). As with both LLTM and MLTM, the general multicomponent latent trait model can be used to (1) test hypotheses about the theoretical variables that underlie response difficulty and (2) estimate parameters that describe test items by basic substantive properties. However, GLTM contains both component outcomes and complexity factors in a single model and may be applied to data that neither LLTM nor MLTM can handle. Joint maximum likelihood estimators are presented for the parameters of GLTM and an application to cognitive test items is described.

Key words: latent trait models, item response theory, aptitude process model.

Both theoretical and practical influences on psychological testing have made identifying the processes, strategies and knowledge structures that are involved in solving test items an important research topic. Theoretically, the information-processing approach in psychology has led to a new type of construct validity research, namely, studies on construct representation (Embretson, 1983). The goal of construct representative research is to identify the theoretical construct(s) that are involved in performance. Task decomposition methods, such as mathematical modeling, have been applied to study construct representation on both cognitive test items (e.g., Carroll, 1976; Pellegrino & Glaser, 1979; Sternberg, 1977) and personality test items (e.g., Cliff, Bradley & Girard, 1973; Cliff, 1977). In contrast, traditional construct validity research has studied only nomothetic span; that is, it examined the relationships of test scores with other measures of individual differences.

Practically, public concern about testing has not been satisfied by traditional validity data, as item disclosure is now legally required in many states. Although the test publishers point to data on criterion related validity, test critics have been more directly concerned with the test item itself and the skills that are involved in its solution. Unfortunately, test developers do not have systematic data on the information processing characteristics of their items.

Two latent trait models have been developed to link item responses to the theoretical variables that influence performance. Both models have been applied mainly to measures of cognitive aptitude. The models are (1) the linear logistic latent trait model (Fischer, 1973) and (2) the multicomponent latent trait model (Whiteley, 1980). The models are similar in that both models are extensions of the Rasch (1960) latent trait model and both models include parameters for underlying response components. However, the models differ in both dimensionality and method of component identification.

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model (GLTM) that combines the linear logistic latent trait model (LLTM) and the multi-component latent trait model (MLTM). The general model may be applied to complex data that neither GLTM or MLTM can handle.

**Two Latent Trait Models for Response Processes**

Both the linear logistic latent trait model and multicomponent latent trait model combine a *mathematical model* for task components with a *latent trait model* (i.e., the Rasch model) for individual differences. The mathematical models link the items to the theoretical component variables, while the latent trait model specifies how person and item parameters combine to specify response likelihoods.

**The Linear Logistic Latent Trait Model**

*The model.* The LLTM is a unidimensional model in which components are identified from item scores on complexity factors that are postulated to determine item difficulty. The model will be presented briefly here. A more comprehensive elaboration of the model is presented in Fischer and Formann (1982).

The model can be examined by considering three equations: The first equation is the mathematical model for task processes. Here, a linear model of the complexity factors, \( c_{im} \), multiplied by their difficulty, \( \eta_m \), predicts item difficulty, \( b^*_i \).

\[
b^*_i = \sum_m c_{im} \eta_m + d
\]

where
- \( c_{im} \) = the complexity of factor \( m \) in item \( i \)
- \( \eta_m \) = the difficulty of complexity factor \( m \)
- \( d \) = a normalization constant

In Equation 1, item difficulty is scaled as a location on the latent ability continuum or trait that determines responses.

The second equation presents the latent trait model for individual differences, which is the Rasch latent trait model,

\[
P(x_{ij} = 1 \mid \theta_j, b^*_i) = \frac{\exp (\theta_j - b^*_i)}{1 + \exp (\theta_j - b^*_i)}
\]

where
- \( x_{ij} \) = the response of person \( j \) to item \( i \)
- \( \theta_j \) = ability for person \( j \)
- \( b^*_i \) = difficulty for item \( i \)

Equation 3 combines these two models to give the linear logistic latent trait model as follows:

\[
P(x_{ij} = 1 \mid \theta_j, \eta_m, d) = \frac{\exp \left( \theta_j - \left( \sum_m c_{im} \eta_m + d \right) \right)}{1 + \exp \left( \theta_j - \left( \sum_m c_{im} \eta_m + d \right) \right)}
\]

If the number of complexity factors equals the number of items, and each item contains only one complexity factor, then LLTM is equivalent to the Rasch latent trait model. LLTM is a linearly constrained model of item difficulty because item difficulty is modeled by a smaller number of factors, \( \eta_m \).

To understand how components are identified, consider the geometric analogy that is presented in Figure 1, which is similar to items on the non-verbal section of the Cognitive