CONDITIONS FOR RASCH-DICHOTOMIZABILITY OF THE UNIDIMENSIONAL POLY TOMOUS RASCH MODEL

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Jansen and Roskam (1986) discussed the compatibility of the unidimensional polytomous Rasch model with dichotomization of the response continuum. They derived a rather strict condition in which dichotomization of multicategory data that fit the unidimensional polytomous Rasch model, results in dichotomous data which fit the dichotomous Rasch model with effectively the same subject parameter. In this paper a more general dichotomization condition is derived for the polytomous Rasch model, which appears less restrictive, but upholds that the intrinsic logic of the unidimensional polytomous Rasch model defies dichotomization in general. The robustness of dichotomous analysis is investigated in a simulation study. It shows a close relation with the two-parameters (Birnbaum) model. Theoretical and methodological implications are discussed.

Key words: latent trait theory, dichotomization, polytomous Rasch model, graded responses, rating data.

1. Introduction

The unidimensional polytomous Rasch model (UPRM, see (1) below) is a model for questionnaire data where the response format consists of graded response categories. Such response categories can be ordered a priori on account of the ordinal qualifiers in their phrasing, for example, "not at all ...", "moderately ...", "strongly ..." and the like. A response format of graded categories can be dichotomized in a natural way, for instance by recoding categories \( g = 1, \ldots, h - 1 \) as "no, disagree", and \( g = h, \ldots, M \) as "yes, agree" (we assume \( M \) response categories, with their indices corresponding to their a priori ordering). The question arises whether polytomous data which satisfy the UPRM would, after dichotomization, satisfy the dichotomous Rasch model, except, possibly, for an admissible transformation of the person- and item-parameters. In that case, the UPRM and the dichotomous Rasch model (RM), are equally valid as psychometric measurement models for the same data, though with some difference in reliability or precision of measurement, due to loss of information by dichotomizing originally polynomous data. The question of the equivalence of UPRM and RM is of practical relevance since difficulties can arise in estimating the parameters of the UPRM when certain response categories are only infrequently used by the subjects in a sample. The
question is also of theoretical interest since the steps in the grading of the response categories are usually more or less arbitrary, and one would like to feel free to replace or combine response categories (in line with their natural semantic ordering) without impairing the validity of person measurement when the data are analyzed by the same general model.

Jansen and Roskam (1986) showed that, in general, the Rasch model for dichotomized data is not compatible with the UPRM for the originally polytomous data. They showed that the equivalence of the models depends in a non-trivial way on the parameter values in the UPRM. Their result is summarized below. It implies that the category parameters in the UPRM interfere with person- and item-measurement, in the sense that combining one set of graded responses into another set of semantically equivalent categories affects person- and item-measurement, unless a rather strict condition is met. For the case of dichotomization, this condition is given in (9) below. In the sequel of this paper, we will derive a more general and less constrained condition which modifies the earlier result of Jansen and Roskam.

2. The UPRM and the RM

In the general graded response case, there are no restrictions on the format of the response categories. The UPRM requires that the same response categories are used for all items.

Apart from the person parameter ($\xi$), representing, for example, the person’s attitude, and the item parameter ($\sigma$) representing the item’s “difficulty” to be endorsed, the UPRM contains two more parameters for each response category. One ($\psi$) represents the “ easiness” or “attractiveness” of the response category, and is usually called a location parameter. The other ($\phi$) weights the difference $\xi - \sigma$ and represents the discriminative power of the category; since it enters into the sufficient statistics to estimate the person- and item-parameters, it is usually called the scoring parameter. The reader is referred to Jansen and Roskam (1986), Andersen (1973, pp. 40-43) and Fischer (1974, pp. 436 seqq.) for details.

In the UPRM, the probability $P_{vik}$ of subject $v$ ($v = 1, \ldots, N$) responding in category $k$ ($k = 1, \ldots, M$) of item $i$ ($i = 1, \ldots, K$) is represented as:

$$P_{vik} = \frac{\exp \{\phi(k)(\xi_v - \sigma_i) + \psi(k)\}}{\sum_{k=1}^{M} \exp \{\psi(g)(\xi_v - \sigma_i) + \psi(g)\}}$$ (UPRM),

where

- $\xi_v$: subject parameter;
- $\sigma_i$: item parameter;
- $\phi(k)$: category scoring parameter (weight of category $k$: the same for all items);
- $\psi(k)$: category location parameter (appeal of category $k$: the same for all items).

In order to be identifiable, some constraints must be imposed on the parameters in (1). Numerator and denominator of (1) can be divided by, for example, $\exp \{\phi(1)(\xi_v - \sigma_i) + \psi(1)\}$, which is equivalent to setting $\phi(1) = \psi(1) = 0$, that is, applying the transformations

$$\phi(g) \rightarrow \phi(g) - \phi(1), \text{ and}$$

$$\psi(g) \rightarrow \psi(g) - \psi(1), \text{ for all } g = 1, \ldots, M,$$