A DYNAMIC GENERALIZATION OF THE RASCH MODEL

N. D. VERHELST AND C. A. W. GLAS

NATIONAL INSTITUTE OF EDUCATIONAL MEASUREMENT (CITO), ARNHEM, THE NETHERLANDS

In the present paper a model for describing dynamic processes is constructed by combining the common Rasch model with the concept of structurally incomplete designs. This is accomplished by mapping each item on a collection of virtual items, one of which is assumed to be presented to the respondent dependent on the preceding responses and/or the feedback obtained. It is shown that, in the case of subject control, no unique conditional maximum likelihood (CML) estimates exist, whereas marginal maximum likelihood (MML) proves a suitable estimation procedure. A hierarchical family of dynamic models is presented, and it is shown how to test special cases against more general ones. Furthermore, it is shown that the model presented is a generalization of a class of mathematical learning models, known as Luce's beta-model.

Key words: Rasch model, missing data, incomplete designs, dynamic models, mathematical learning theory.

Introduction

Perhaps the most outstanding feature of IRT models is that interindividual differences in behavior are explained at the model level, implying that subjects need not be considered as statistical replicates of each other, and it is not necessary to sample multiple observations from the same subject under assumedly constant conditions. This is accomplished by introducing so-called incidental parameters, one parameter associated with each individual. Although these parameters can be treated formally as any other parameter in the model, a major problem is associated with their presence in parameter estimation. Maximizing the likelihood, for example, will not yield consistent estimates of the incidental parameters in general, nor of the other (sometimes called structural) parameters of the model (Neyman & Scott, 1948). There are several ways to cope with this problem. Two estimation methods are often used in IRT. In the first method, sufficient statistics for the incidental parameters (if they exist) are considered constants, and the conditional likelihood, given these statistics, is maximized. This conditional likelihood is by definition independent of the incidental parameters, and Andersen (1973) proved that this method yields consistent estimates of the structural parameters. This method is commonly labeled conditional maximum likelihood (CML). In the other approach, the incidental parameters are no longer considered fixed constants, but are treated as realizations of a (nonobserved or latent) random variable, whose distribution is assumed to belong to a certain family of distributions. In many applications this family is parameterized by a finite number of parameters (for example, the family of normal distributions). Since the latent variable is not observed, it is integrated out from the likelihood function, yielding the so-called marginal likelihood function. This function is maximized with respect to the structural model parameters and the parameters of the distribution jointly, yielding marginal maximum likelihood (MML) estimates, which by a seminal paper of Kiefer and Wolfowitz (1956) are proved...
to be consistent under very mild regularity conditions. The use of CML is restricted to models that have minimal sufficient statistics for the incidental parameters, and among the IRT models commonly applied very few are in this class (see Verhelst & Eggen, 1989, for a general characterization). The most famous example is the Rasch model, and both estimation methods have been successfully applied to this model (for CML, see Fischer, 1974; and Rasch, 1960; for MML, see Glas, 1989; and Thissen, 1982).

Several authors (Fischer, 1981; Mislevy, 1984) noted the possibility of obtaining consistent estimates of the structural parameters in the Rasch model from partially incomplete data or data collected in an incomplete design. It is the purpose of the present paper to investigate the applicability of the Rasch model in a dynamic context by manipulating the missing data concept on a set of complete data. This manipulation is convenient to get around the restrictions implied by the central axiom of local stochastic independence common to most latent trait models. For one thing, this axiom implies that a correct response by a given subject is independent of the responses on previous items or trials, thus seemingly excluding the so-called subject controlled learning models where the probability of a correct answer depends explicitly on the response pattern given thus far (Sternberg, 1963). Although there exist generalizations of the Rasch model, where this dependence can be modeled explicitly (Jannarone, 1986; Kelderman, 1984), parameter estimation is difficult and limited to cases with a rather restricted number of parameters. Special mention needs to be made of the dynamic test model by Kempf (1974, 1977a, 1977b), which, in the sequel, will be contrasted with the model of the present paper. The approach used here combines the Rasch model with the missing data concept and with linear restrictions on the parameter space, yielding a wide range of dynamic models. The basic approach consists in conceiving of an item (or trial in a learning experiment) as a collection of "virtual" or "technical" items, one of which is supposed to be administered to each subject, depending, for example, on the pattern of previous responses. This is possible because of the basic symmetry in the Rasch model, where a change in the latent ability can equally well be considered a complementary change in the item difficulty. The details of this approach are the subject matter of the next section.

The idea presented here is not new; in fact, it has been considered by Fischer (1972, 1983). Working in the context of CML estimation, Fischer rejected this approach because the model thus constructed was not estimable. In the present paper, it will be shown that the models constructed are in general estimable under MML and statistical tests of special cases of the model against more general ones can easily be constructed. Finally, it will be shown that a class of mathematical learning models, known as Luce's beta model and generalizations thereof, are a special case of our model.

The Dynamic Rasch Model

In mathematical learning theory (see Sternberg, 1963, for a general introduction), the control of change in behavior is generally attributed to two classes of events: one is the behavior of the responding subject itself; the other comprises all events that occur independently of the subject's behavior, but which are assumed to change that behavior. Models that only allow for the former class are called "subject controlled"; if only external control is allowed, the model is "experimenter controlled", and models where both kinds of control are allowed are labeled "mixed models". As an example of "experimenter control", assume that during test taking, the correct answer is given to the respondent after each response. If it is assumed that learning takes place under the influence of feedback (generally referred to as reinforcement) independent of the correctness of the given response, the model is experimenter controlled; if it is assumed,