A GENERALIZATION OF TAKANE’S ALGORITHM FOR DEDICOM

HENK A. L. KIERS AND JOS M. F. TEN BERGE
UNIVERSITY OF GRONINGEN

YOSHI TAKANE
MCGILL UNIVERSITY

JAN DE LEEUW
UNIVERSITY OF CALIFORNIA, LOS ANGELES

An algorithm is described for fitting the DEDICOM model for the analysis of asymmetric data matrices. This algorithm generalizes an algorithm suggested by Takane in that it uses a damping parameter in the iterative process. Takane’s algorithm does not always converge monotonically. Based on the generalized algorithm, a modification of Takane’s algorithm is suggested such that this modified algorithm converges monotonically. It is suggested to choose as starting configurations for the algorithm those configurations that yield closed-form solutions in some special cases. Finally, a sufficient condition is described for monotonic convergence of Takane’s original algorithm.

Key words: DEDICOM, least squares fitting, majorization.

DEDICOM is a model proposed by Harshman (1978) for the analysis of asymmetric data. For an extensive description of this model we refer to Harshman, Green, Wind, and Lundy (1982). A brief description of the model will be given here. According to the DEDICOM model a square data matrix \( X \), containing entries \( x_{ij} \) representing the (asymmetric) relation of object \( i \) to object \( j \) is decomposed as

\[
X = ARA' + N,
\]

(1)

where \( A \) is an \( n \) by \( p \) (\( p < n \)) matrix of weights for the \( n \) objects on \( p \) dimensions or aspects, \( R \) is a square matrix of order \( p \), representing (asymmetric) relations among the \( p \) dimensions, and \( N \) is an error matrix with entries \( n_{ij} \) representing the part of the relation of object \( i \) to object \( j \) that is not explained by the model. The objective of fitting this model to the data is to explain the data by means of relations among as small a number of dimensions as possible. These dimensions can be considered as "aspects" of the objects. The "loadings" of the objects on these aspects are given by matrix \( A \). The entries in matrix \( A \) indicate the importance of the aspects for the objects. The dimensionality of \( R \) and \( A \), and hence the number of aspects to be determined, is to be based on some external criterion, defined by the user.

Several algorithms have been developed for fitting the DEDICOM model. A comparison of most of these has been given by Harshman and Kiers (1987). Kiers (1989) has discussed properties of a number of these algorithms and concludes that his column-wise alternating least squares algorithm is preferable from various points of view.

Financial Support by the Netherlands organization for scientific research (NWO) is gratefully acknowledged. The authors are obliged to Richard Harshman.

Requests for reprints should be sent to Henk A. L. Kiers, Department of Psychology, Grote Markt 31/32, 9712 HV Groningen, THE NETHERLANDS.
One of the methods discussed by Harshman and Kiers (1987) is a method proposed by Takane (1985). His method appeared to be very efficient in most practical cases, but turned out to be inadequate in some cases. Moreover, no general convergence properties are known for this method. Therefore, this method has not been recommended for general use. In the present paper, it will be shown that a slight modification of Takane’s method is sufficient to overcome these problems.

Before describing the resulting modified algorithm, a brief description of the DEDICOM model will be given. Next, a new type of algorithm will be discussed from which Takane’s method can be derived as a special case. Finally, it will be described how Takane’s method is to be modified in order to obtain an efficient algorithm that does converge monotonically.

### A Monotonically Converging Algorithm for DEDICOM

The DEDICOM model has to be fit in the least squares sense over matrices $A$ and $R$ of order $n$ by $p$, and $p$ by $p$, respectively. Without loss of generality matrix $A$ is constrained to be column-wise orthonormal. The loss function that is to be minimized can be written as

$$\sigma(A, R) = \|X - ARA'\|^2. \quad (2)$$

Because $A'A = I_p$, the minimum of $\sigma$ over $R$ for fixed $A$ is given by $R = A'XA$. Minimizing (2) over $A$, for fixed $R$, is equivalent to maximizing

$$f(A) = \text{tr} A'XAA'X'A, \quad (3)$$

over matrix $A$, subject to the constraint $A'A = I_p$. The algorithm to be presented here is based on the following results.

**Result 1.** Let $X$ and $A$ be fixed matrices of appropriate orders, and let $E$ be any matrix of the same order as $A$, then we have

$$\text{tr} E'XEA'X'A \geq -\alpha \text{ tr } E'E, \quad (4)$$

if $\alpha$ is some scalar not smaller than the largest eigenvalue of the symmetric part of $(-X \otimes A'XA)$, where $\otimes$ refers to the Kronecker product of matrices.

**Proof.** In order to prove (4) we rewrite the left hand side of (4) as follows. Let $e$ denote $\text{Vec}(E)$, the vector with the elements of $E$ strung out row-wise into a column-vector, then

$$\text{tr} E'XEA'X'A = -\text{tr} E'(-X)EA'X'A = -e'(-X \otimes A'XA)e. \quad (5)$$

It should be noted that $(-X \otimes A'XA)$ is not generally symmetric.

As is readily verified, for any square matrix $C$ of appropriate order, we have

$$e'Ce = e'[\frac{1}{2} (C + C')]e = e'C_s e, \quad (6)$$

where $C_s$ denotes the symmetric part of $C$. Let $\alpha$ be the largest eigenvalue of $C_s$, then it is well-known that

$$e'C_se \leq \alpha e'e. \quad (7)$$

Combining (6) and (7) we have

$$e'Ce \leq \alpha e'e. \quad (8)$$