THE MAGNETOSPHERE AS A NONLINEAR SYSTEM

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Summary: In order to study the nonlinear physical processes connected with substorm activity we analyse time series of local geomagnetic field variations. The concepts of deterministic chaos and magnetospheric chaotic attractors are examined. The general objective of this article is to detect low dimensional magnetosphere chaos and to properly interpret it as a consequence of magnetosphere-ionosphere informational-energetic coupling.

1. INTRODUCTION

In the past few decades one has seen remarkable achievement in the application of linear modelling in such diverse fields as physics, biology, chemistry, ecology, etc. These achievements are perhaps rather natural in view of the significant contributions of linear differential equations in all branches of science. It may be said that the basic idea underlying the linear analysis is the principle of superposition. As every real system is nonlinear, an excessive adherence to this principle would become drawback.

Strictly speaking, any system of linear differential equations is totally inadequate as a tool to analyse more intricate phenomena such as the rising of natural macroscopic structures, their evolution and survival, turbulence, phase transitions, time irreversibility, coexistence of species, morphogenesis, and thinking. This list refers only to some examples of intricate physical phenomena.

There is no doubt that linear modelling should be replaced by a much wider and deeper nonlinear class of analysis of dynamical systems. Since a large variety of complex natural systems has attracted scientific attention, the interdisciplinary character of such research is quite evident. Most of our recent knowledge of complex dynamical systems originates in diverse fields of scientific research. However, as has been demonstrated in various cases, it is possible to formulate universal nonlinear laws which are valid for many complex systems. Lorenz's deterministic chaos [1], Prigogine's irreversible thermodynamics [2] and Haken's synergetics [3–5] were the first and successful attempts to understand nonlinear macroscopic phenomena.

The aim of this paper is to show that geomagnetic storm-time variations of apparently stochastic character possess deterministic features. Due to nonlinear magnetosphere-ionosphere interactions magnetosphere deterministic chaos should arise as a low dimensional phenomenon. We make an effort to interpret magnetosphere chaos at the qualitative level. Our suggestion is that coherent space-time structures within the magnetosphere-ionosphere complex system correspond to a truncated system of differential equations of unstable modes of motion, deduced by the synergetic slaving principle [5]. This means that, due to nonlinear mode competition, a very high number of degrees of freedom become enslaved and the macroscopic dynamics and structures will be determined only by a few degrees of freedom. The physical meaning of the slaving principle is based on strong dissipation of energy of the enslaved modes of motion. Our

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The Magnetosphere as a Nonlinear System

expectation is that the synergetic approach combined with information theoretical description [6,7] should answer many questions connected with the appearance of ionospheric, auroral, magnetopause, and other coherent space-time structures on a given scale, however, it would not explain fully developed turbulence, stochastic motions, etc., [8].

In the following parts of this paper the concepts of deterministic chaos and chaotic attractors [1] will be widely used. Therefore, we describe the basic notions used in the analysis of chaotic states in the next section.

2. CHAOTIC ATTRACTORS

As usual, we assume that the time evolution of a dynamical system can be described by a differentiable dynamical system in a phase space of possible infinite dimensions. Due to strong dissipation a finite dimensional chaotic or strange attractor corresponds in many cases to a chaotic state. A closed set $A$ is called an attractor if, for $t \to \infty$, all near-by trajectories in phase space tend to $A$. Strange attractors are typically characterized by fractal dimensionality, while the term "chaotic" refers to the Lyapunov instability of phase trajectories. The details of the temporal evolution of phase trajectories on a chaotic attractor can be quantitatively characterized by the spectrum of Lyapunov exponents. In general, the Lyapunov exponents, $\lambda_i$, may be calculated by linearizing the relevant equations and studying the evolution of small perturbations $w$, as shown by Haken [4] and Wolf et al. [9].

Then

$$\lambda_i = \lim_{t \to \infty} \sup \left\{ \frac{1}{t} \ln |w_i(t)| \right\} , \quad i = 1, 2, ..., N$$

(1)

If $\lambda_i > 0; \ (i = 1, 2, ..., j; j \leq N)$, the motion along trajectories is unstable and small perturbations grow exponentially with time. This phenomenon of an error's exponential growth is called a sensitive dependence on the initial conditions. The cases in which all $\lambda_i \leq 0$ correspond to stationary or periodic states.

A chaotic (or strange) attractor can be quantitatively characterized by its metric properties, by means of static, time independent invariants. For this purpose the dimension spectrum of attractors has been defined [6] as

$$D^{(k)} = \frac{1}{k - 1} \lim_{\epsilon \to 0} \frac{\ln \sum_{i=1}^{m} P_i^k}{\ln \epsilon} ,$$

(2)

where $P_i$ is the probability that a point on a trajectory falls into the $i$-th box of size $\epsilon$ in phase space partitioned into $m$ hypercubes. The most commonly used dimension $D^{(2)}$ is the so-called correlation dimension $D^{(2)}$ which gives a lower estimate of the minimum number of variables (degrees of freedom) which are at least required to describe the system.

Naturally, a unique possibility is offered to answer the question whether the dynamics of a complex system is low-dimensional. A finite correlation dimension indicates that,