STATIONARY APPROXIMATION OF THE MAGNETOTELLURIC FIELD IN A HALFSPACE WITH A VERTICAL BOUNDARY AND THREE-DIMENSIONAL PERTURBING BODY

MILAN HVOŽDARA

Geophysical Institute, Slovak Acad. Sci., Bratislava*)

Summary: Based on the generalized potential of a double layer, integral formulae have been derived for calculating the stationary approximation of the magnetotelluric field in a halffspace divided by a vertical boundary into two quarterspaces, one of which contains a three-dimensional perturbing body. The appropriate boundary integral equation and other surface integrals have been computed for a perturbing body in the shape of the three-dimensional prism located at the vertical boundary, or in contact with the said boundary. The exciting electrical field is assumed to be homogeneous and perpendicular to the vertical boundary. Isoline graphs of the electrical and magnetic fields on the surface of the halffspace have been plotted and their anomalies are discussed.

1. INTRODUCTION

It is important to know the solution of the direct problem of geoelectricity for a number of models with lateral inhomogeneous distribution of the electrical conductivity for the purposes of geoelectrical methods used in geophysics. One of these is also a conducting halffspace $z > 0$, divided by a vertical plane boundary (the plane $x = 0$) into two quarterspaces, one of which ($x > 0$, region "1") contains

![Diagram of the 3D perturbing body in the neighbourhood of the vertical boundary of two quarterspaces. The distances $R, R_+, R_x, R_x^+$ occurring in Green's functions of the integral formulae, are also shown.](image)

Fig. 1. Diagram of the 3D perturbing body in the neighbourhood of the vertical boundary of two quarterspaces. The distances $R, R_+, R_x, R_x^+$ occurring in Green's functions of the integral formulae, are also shown.

*) Address: Dúbravská cesta 9, 842 28 Bratislava.
Stationary Approximation of the Magnetotelluric Field...

a three-dimensional perturbing body, a conductivity inhomogeneity (Fig. 1). In quarterspace \( x < 0 \) (region "2") the electrical conductivity is assumed to be \( \sigma_2 \), in quarterspace "1" \( \sigma_1 \) and in the perturbing body \( \sigma_T \). In the absence of the perturbing body, the external sources of the exciting electrical field are assumed to generated potential \( V_1 \) in quarter space \( x > 0 \) and \( V_2 \) in quarterspace \( x < 0 \). These potentials are assumed to be known (given). At points outside the sources \( V_1 \) and \( V_2 \) satisfy Laplace's equation:

\[
\nabla^2 V_1 = 0, \quad \nabla^2 V_2 = 0.
\]

At the boundary \( z = 0 \) the well-known boundary conditions

\[
\left[ \frac{\partial V_1}{\partial z} \right]_{z=0} = 0, \quad \left[ \frac{\partial V_2}{\partial z} \right]_{z=0} = 0
\]

apply, because this is a boundary of a conducting halfspace with a non-conducting medium (air). The following conditions must hold at the vertical boundary \( x = 0 \):

\[
\left[ V_1 \right]_{x=0} = \left[ V_2 \right]_{x=0}, \quad \sigma_1 \left[ \frac{\partial V_1}{\partial x} \right]_{x=0} = \sigma_2 \left[ \frac{\partial V_2}{\partial x} \right]_{x=0}.
\]

These potentials can be determined relatively easily using classical methods of solving geoelectrical problems, of approximating fields of magnetotelluric currents, or for fields excited by point electrodes.

2. INTEGRAL EXPRESSION OF THE SOLUTION FOR THE INHOMOGENEOUS CASE

Let \( S \) be the surface of the perturbing body in quarterspace "1", \( V_T \) its volume and \( \sigma_T \) its electrical conductivity. As the result of the presence of this perturbing body, potentials \( V_1 \) and \( V_2 \) change and, consequently, we shall denote the resultant potential in "1" as \( U_1 \) and in "2" as \( U_2 \). Within the body assume the potential to be \( U_T \). All these potentials are harmonic functions outside the points with the sources of the exciting field, i.e. they satisfy Laplace's equation

\[
\nabla^2 U = 0 \quad \text{for} \quad U = U_1, U_2, U_T.
\]

At boundary \( z = 0 \) the following boundary conditions must hold:

\[
\left[ \frac{\partial U_k}{\partial z} \right]_{z=0} = 0, \quad k = 1, 2.
\]

The following must also hold at the vertical boundary \( x = 0 \):

\[
\left[ U_1 \right]_{x=0} = \left[ U_2 \right]_{x=0}, \quad \sigma_1 \left[ \frac{\partial U_1}{\partial x} \right]_{x=0} = \sigma_2 \left[ \frac{\partial U_2}{\partial x} \right]_{x=0}.
\]

On the surface \( S \) of the perturbing body, the following boundary conditions apply:

\[
\left[ U_1 \right]_S = \left[ U_T \right]_S, \quad \sigma_1 \left[ \frac{\partial U_1}{\partial n} \right]_S = \sigma_T \left[ \frac{\partial U_T}{\partial n} \right]_S,
\]

where \( n \) is the unit vector of the outward normal to \( S \). This normal is assumed to be