ON ONE METHOD OF SOLVING THE TWO-DIMENSIONAL DIRECT MAGNETIC PROBLEM

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Summary: In this paper the solution of the direct magnetic problem for two-dimensional bodies, founded on the application of Green's theorem is derived. This solution is derived under the assumption that the components of the magnetization vector have continuous derivatives with respect to the coordinates and that they are continuous within the body. The problem is solved in terms of Green-type integrals for the scalar and vector potential of the magnetostatic field and it may serve the purpose of solving the problem of the analytical continuation of the external field into the body.

1. INTRODUCTION

The solutions of the direct magnetic problem for two-dimensional bodies were derived in the form of Cauchy-type integrals in [1, 2], where methods of the theory of the complex variable were used for this purpose. The method based on Green's formula was used for solving the two-dimensional direct gravity problem in [3] where it was shown that this method is a good supplement to the methods of solving these problems using the theory of the function of the complex variable and Cauchy-type integrals. The objective of this paper is to show that this method can also be used for solving the direct magnetic problem for two-dimensional bodies.

2. FORMULATION AND SOLUTION OF THE PROBLEM

To be able to give the solution of the direct magnetic problem in the form of Green-type integrals, it is necessary to modify the well-known Green's formula for a plane to other forms more useful for this purpose.

Consider S to be a simply connected domain bounded by a piece-wise smooth and closed curve s. Now by S' we will understand the domain external to domain S. The outward normal to this curve is assumed to be n. Point Q(ξ₁, ξ₂) is a variable point of the domain S + s, point P(x₁, x₂) is the point of observation, (ξ₁, ξ₂) and (x₁, x₂) are the Cartesian coordinates of these points. We will consider the vector \( \mathbf{G} = G(Q) \).

Assume that the components of this vector \( G_i \), \( i = 1, 2 \) are continuous functions with continuous first derivatives with respect to the coordinates \( ξ_i \), \( i = 1, 2 \) at each point \( Q \in S + s \) and that they have continuous second derivatives in domain S. These functions \( G_i = G_i(Q) \), \( i = 1, 2 \) satisfy the condition of Green's formula.

Now we can apply Green's formula in the plane derived, e.g., in [4] to functions \( G_i = G_i(Q) \) and \( v = \ln \frac{1}{R} \). By R we will understand the distance between points Q and P: \( R = [(x_1 - ξ_1)^2 + (x_2 - ξ_2)^2]^{1/2} \). For an exterior point \( P \in S' \) it then holds that

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On One Method of Solving the Two-dimensional Direct Magnetic Problem

\[\frac{1}{2\pi} \oint_S \left[ \ln R \frac{\partial G_i}{\partial n} - G_i \frac{\partial \ln R}{\partial n} \right] ds = \frac{1}{2\pi} \oint_S \ln R \Delta G_i \, dS, \quad (i = 1, 2), \quad P \in S'. \]

Now let us apply the differential operator \( \text{grad} \) with respect to the coordinates of the exterior point \( P \). Provided the functions under the integrals are continuous and have continuous derivatives, operator \( \text{grad}_p \) can be transferred under the integral sign. Operator \( \text{grad}_p \) operates under the integral on the product of the functions, where functions \( G_i, (i = 1, 2) \) are functions of the variable point \( Q \), and that is why they behave as constants with respect to the operator. Owing to the continuity of function \( \ln R \) on \( s \), it is possible to exchange the order of the normal derivative for operator \( \text{grad}_p \). If the radius-vector \( R \) is oriented in the direction from point \( Q \) to point \( P \), then \( \text{grad}_p (\ln R) = R/R^2 \). Equations (1) then yield

\[\frac{1}{2\pi} \oint_S \left[ \frac{R}{R^2} \frac{\partial G_i}{\partial n} - G_i \frac{\partial}{\partial n} \left( \frac{R}{R^2} \right) \right] ds = \frac{1}{2\pi} \oint_S \Delta G_i \frac{R}{R^2} \, dS, \quad (i = 1, 2). \]

After scalar producting equations (2) by vectors \( e_i \), where \( e_i \) are unit vectors in the direction of axes \( x_i (i = 1, 2) \) of the Cartesian coordinate system and, further, after adding these equations, taking into account that

\[\sum_{i=1}^{2} \frac{\partial G_i}{\partial n} e_i = \frac{\partial \mathbf{G}}{\partial n}; \quad \sum_{i=1}^{2} G_i e_i = \mathbf{G}; \quad \sum_{i=1}^{2} \Delta G_i e_i = \Delta \mathbf{G}, \]

Eq. (2) yield the following equation

\[\frac{1}{2\pi} \oint_S \left[ \frac{R}{R^2} \frac{\partial \mathbf{G}}{\partial n} - \mathbf{G} \cdot \frac{\partial}{\partial n} \left( \frac{R}{R^2} \right) \right] ds = \frac{1}{2\pi} \oint_S \frac{\Delta \mathbf{G} \cdot R}{R^2} \, dS, \quad P \in S'. \]

After vector producting equations (2) by unit vectors \( e_i \) and after adding these equations, with a view to (3) it then holds that

\[\frac{1}{2\pi} \oint_S \left[ \frac{\partial \mathbf{G}}{\partial n} \times \frac{R}{R^2} - \mathbf{G} \times \frac{\partial}{\partial n} \left( \frac{R}{R^2} \right) \right] ds = \frac{1}{2\pi} \oint_S \frac{\Delta \mathbf{G} \times R}{R^2} \, dS, \quad P \in S'. \]

Assume point \( P \) to be the internal point \((P \in S)\). After applying Green’s formula in the plane to functions \( G_i, (i = 1, 2) \) and \( v = \ln 1/R \), we obtain for point \( P \in S \)

\[\frac{1}{2\pi} \oint_S \left[ \ln R \frac{\partial G_i}{\partial n} - G_i \frac{\partial}{\partial n} (\ln R) \right] ds = -G_i(P) + \frac{1}{2\pi} \oint_S \Delta G_i \ln R \, dS, \quad (i = 1, 2), \quad P \in S'. \]

Now we will repeat the procedure with the application of the differential operator \( \text{grad}_p \) to Eqs (6) and we will use the same mathematical operation as in the case of the external point. In view of