NON-HYDROSTATIC MODEL OF AIRFLOW OVER IRREGULAR TOPOGRAPHY – THEORETICAL BASES

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Summary: This article deals with some problems connected with the formulation of a non-hydrostatic mesoscale model of airflow in the atmosphere. Due to an irregular surface a terrain-following coordinate system is used and the equations of the model are transformed into this system. Sound waves are eliminated by the use of the anelastic approximation. The influence of boundaries is minimized by the use of open boundary conditions at the lateral boundaries of the computational domain and of the absorbing layer beneath the upper boundary.

1. INTRODUCTION

A number of specific features of atmospheric circulation are related to mesoscale processes. These processes cannot be described usually by means of global circulation models because the characteristic scales of mesoscale processes are smaller than the resolution of the global forecast model. This fact has led to research and then to the operational use of special mesoscale models (see for instance [9]).

Although there are different views of the problem of specifying characteristic space and time scales of mesoscale processes, it can be said that the magnitude of the characteristic space scale lies in the range of a few kilometres (meso $7$ scale) to a few hundreds of kilometres (meso $z$ scale) and the time interval, characterizing the duration of a typical mesoscale phenomenon ranges from a few hours up to one day (see for instance [2]).

On the basis of the preceding facts one could say that a proper mesoscale model could be obtained with a refinement of a grid mesh of some global forecast model. But the reality is rather different and more complicated. One must bear in mind that a greater number of factors influencing the behaviour of the atmosphere must be taken into account than in case of global models. It is required that a large number of processes not involved into a global model or only roughly parametrized be described explicitly or be parametrized to a higher degree of approximation. Another important difference results from the fact that a mesoscale model is limited in area and thus it covers a much smaller area than the global model. The following questions, which are consequences of the problems mentioned above, deal with the use of a hydrostatic or non-hydrostatic approximation, with the method of formulating boundary conditions or with the choice of an adequate coordinate system, in which the problem is solved; the last item is related both to the decreasing value of the vertical grid step and, above all, to more detailed description of the orography. These questions can be encountered in a number of studies.

Problems dealing with open lateral boundary conditions are treated for instance in [4], [11] or [12] and those dealing with the appropriate expression of the upper boundary conditions are analyzed in [1], [3], [5] or [6]. Whether to use a hydrostatic or a non-hydrostatic approximation in mesoscale models was discussed by Tag and Rossmond in [15].

The influence of the earth's surface on atmospheric processes is investigated in many articles; one can choose among them according to the use of the hydrostatic or non-hydrostatic approximation (e.g. [3], [5] or [10]).

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2. COORDINATE SYSTEMS OF MESOSCALE MODELS

Due to the decreasing length of the vertical grid step and more detailed description of orography the choice of an adequate coordinate system used in mesoscale models is very important. From this point of view the use of a $p$-system or $z$-system of coordinates seems to be insufficient due to the great problems which can arise in both of these coordinate systems in formulating the lower boundary conditions. Similar difficulties can be expected if an isentropic coordinate system is used. Moreover, when the potential temperature is used as the vertical coordinate, the condition of the monotonic decrease or increase with height is not guaranteed. Besides the above mentioned coordinate systems there are those which take into account the variability of the earth's surface. In such systems this surface becomes the lowest coordinate level $z = \text{const}$. The $\sigma$-coordinate system, which has already been "classical", can be mentioned as an example of this type of coordinates. Another possibility of incorporating the shape of orography is the use of the following expression of the vertical coordinate

$$\tilde{z} = z + z_o(x, y), \quad (1)$$

where $\tilde{z}$ is the transformed vertical coordinate, $z$ is the cartesian coordinate and $z_o$ approximates the shape of the earth's surface.

A useful method of expressing the vertical coordinate is the functional form called terrain-following coordinate system or $\sigma$-representation of $z$-system of coordinates (see for instance [14]). This coordinate system is based on the following transformation formulae

$$\xi = x, \quad (2)$$
$$\eta = y,$$
$$\tilde{z} = \frac{H(z - z_o(x, y))}{H - z_o(x, y)},$$

$H$ is the geometrical height of the upper boundary of the model domain in the cartesian system of coordinates $(x, y, z)$ and $(\xi, \eta, \tilde{z})$ are the transformed coordinates.

The relations between the various kinds of coordinate systems are shown in Fig. 1, which has been adopted from [14]. In [14] there is a comprehensive chapter on the questions dealing with various types of coordinate systems.

The use of the coordinate system described by formulae (2) on the one hand causes some complications in expressing the model equations on the other hand the boundary conditions, especially at the bottom boundary, are defined more precisely.

3. BOUNDARY CONDITIONS

Each mesoscale model is a limited-area model and that is why it is necessary to formulate space boundary conditions. These conditions must be consistent with the