Labor Supply Under Uncertainty: A Comment and Extension

ERIC J. SOLBERG*

In a recent short paper, Hartley and Revanker [2] presented a micro-model of the individual's labor supply decision with uncertainty introduced through the unemployment rate. The model is a significant contribution to the theory of labor supply within the context of income-leisure analysis. However, the H-R formulation is somewhat misleading since it first defines $U_{ni}$ as "... the probability that individual $i$ will find himself unemployed..." and then switches to "... the unemployment rate $U_{ni}$." Clearly, the unemployment rate does not measure the probability of being unemployed or not finding a job. However, as the unemployment rate increases or decreases, the probability of employment will decrease or increase. It is this inverse variation between the probability of employment and the unemployment rate that is important to capture in any empirical investigation.

The unemployment rate can be used as a proxy for the probability of unemployment, a poor one, but better than excluding the influence of the probability of unemployment entirely.¹ The use of a poor proxy for an explanatory variable that should be included in an empirical specification is better than none when the properly excluded variable is not measurable [8]. H-R’s contention that the unemployment rate is a legitimate explanatory variable in the structural labor supply equation is not properly supported by the inclusion of the unemployment rate in the budget constraint. The probability of unemployment (or employment) is properly included and the unemployment rate can be used as a proxy in the absence of the ability to measure that probability.

On the basis of their analysis, H-R offer the following propositions [2; pp. 173, 175]:

1. "If the individual maximizes his expected utility, his labor supply curve depends on real wages, real unemployment compensation, real nonwage income, and his own unemployment rate."

2. "The individual's labor supply varies directly with real wages and the unemployment rate, and inversely with real unemployment compensation and real income."

3. "The higher the level of the unemployment rate $U_{ni}$, the greater the rate of response of the individual's labor supply $N_t$ to a change in the unemployment rate."

Certain elements of these propositions seem to be contradicted when an alternative model using slightly different but equally familiar methodology is constructed.

The extension below incorporates a stochastic wage dependent on the probability of employment and considers alternative wage rates as well as unemployment benefits. Further, the standard model of labor supply derived from income-leisure analysis is shown to be a special case of the stochastic wage model when the probability of employment is certain and there are no

* California State University, Fullerton. This paper is based in part on the author's doctoral dissertation [6].

¹ Samuel A. Rea, Jr. has recently reported some empirical results where reported unemployment was included as an explanatory variable in the specified labor supply. Rea divides total leisure into pure leisure and the part of the time spent looking for work in a static income-leisure format under the assumption that part of unemployment represents job-search.
alternative wage rates. No issue will be taken with the H-R propositions 1 and 3 if in place of “own unemployment rate” the “probability of employment” is substituted. However, it will be shown that if the labor supply curve under certainty is positively sloped, an increase in uncertainty will cause the labor supply curve as a function of the real market wage rate to shift to the left. The existence of unemployment benefits and alternative wage rates mitigates this shift in the supply curve.

Section I introduces the stochastic wage through a particular distributional form. The stochastic wage model will be shown to reduce to the certainty model when no alternative wage rates exist and the probability of employment is certain. Section II considers the probability of employment, unemployment benefits, and alternative wages. A series of propositions will be made on the basis of the analysis. Section III offers a rebuttal to potential critique of the stochastic wage model.

Section I.

A. The Stochastic Wage. When an individual is offering to supply labor time, he must consider the probability that he will be unemployed. If \( \pi \) is the probability of employment and \( 1 - \pi \) the probability of unemployment, he will be confronted with the following distribution of the effective wage rate, \( r \):

\[
f(r) = (1 - \pi)f_1 + \pi f_2
\]

where \( f_1 \) is the probability density function (p.d.f) which holds under conditions of unemployment and \( f_2 \) is the p.d.f. which holds under conditions of employment.

A reasonable representation for \( f_1 \) is the discrete p.d.f.

\[
f_1 = \delta(r - r^*), r \geq 0; r^* \geq 0
\]

where \( r^* \) is the unemployment benefits rate measured in units comparable to the wage rate. The p.d.f. \( f_2 \) is likely to be continuous beyond the minimum acceptable wage rate that induces participation in the labor force \( (r^0) \), which might be considered as the legislated minimum wage rate, with frequencies of higher alternative wage rates declining beyond the mode. A function which is intuitively appealing is

\[
f_2 = \Gamma(\beta)^{-1}(1/\alpha)^{-\beta}(r - r^0)^{\beta-1}e^{-\alpha(r-r^0)}, r \geq r^0.
\]

Thus \( f(r) \) is a weighted sum of the discrete p.d.f. with the gamma p.d.f.\(^2\) If \( \beta = 1 \), then \( f_2 \) is the exponential. Figure I pictures \( f(r) \) when the parameter \( \beta \) is constrained to be greater than or equal to unity. The parameter \( \alpha \) can be chosen such that the frequency of some high alternative wage rate is small, since there is little empirical difference between

\[f_2 = \Gamma(\beta)^{-1}(1/\alpha)^{-\beta}(r - r^0)^{\beta-1}e^{-\alpha(r-r^0)}, r \geq r^0.\]

\(^2\) The symbol \( \delta \) denotes the Dirac delta, or unit impulse function, which is defined as a function \( \delta(t) \) such that

\[
\delta(t) = 0 \text{ for } t \neq 0, \int_{-\infty}^{\infty} \delta(t) \, dt = 1;
\]

therefore,

\[
\int_{-\infty}^{\infty} \delta(t)f(t) \, dt = f(0).
\]

See [3, pp. 44–56] and [5, p. 286].

\(^2\) This particular distributional form was suggested by Dr. Donald Ebbeler, Claremont Graduate School.