Simple Search Games on an Infinite Circular Cylinder

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Abstract. Differential search games with simple motions of objects on an infinite circular cylinder are approached geometrically, using sets whose structure depends on time. Sufficient conditions for detection with the corresponding strategies are given. New kinds of problems are proposed, their connections with the initial parameters are investigated.

§1. Introduction

It is generally agreed that the mathematical theory of search was founded in the articles [1-3], which stimulated numerous publications discussing diverse search problems (detailed references can be found in [4-6]). It should be noted that most of the investigations dealt with search problems in which the searching object possessed sufficiently ample information about the location of the eluding object.

The geometric method (the method of tracing areas) used in this work and based on the use of certain auxiliary sets that change in time is one of the approaches that have been developed just recently ([7-11]), although some elements of the geometric approach can be found in [12].

§2. Statement of the search problem

Here we consider the search problem in the following setting. Point objects $A$ (the searching object) and $B$ (the eluding object) move in an arcwise connected set $X$; their speeds are constant and equal to $\alpha$ and $\beta$ ($\beta < \alpha$), respectively. The eluding object $B$ is considered detected if its distance from the searching object $A$ is not greater than a certain given positive $l$. It is assumed that the parameters $\alpha$, $\beta$ and $l$ and the structure of the "hunting ground" $X$ are known to both objects; in addition, the object $B$ also knows the entire trajectory of the object $A$ and the location of $A$ on this trajectory at any time.

The actual problem is to specify

1. sufficient conditions on $\alpha$, $\beta$, and $l$ that make successful search (detection) feasible;
2. a trajectory such that, moving along it, the searching object $A$ will successfully detect $B$.

Remark. It is convenient to assume that at any moment of time the object $A$ is at the center of a circle of radius $l$ forbidden for $B$. This circle will be called the control $l$-circle or just the control circle.

§3. Tracing area

In [7-11], it was shown that the investigation of search problems in the setting described above quite naturally leads to a class of sets of variable structure containing the control $l$-circle and free of the eluding object $B$. These auxiliary sets, called tracing areas are constructed from a given trajectory of $A$ and combine two components, namely the set of points that the eluding object $B$ has no time to reach at a given moment of time (the aftermath area) and the set of points that $B$ has no time to leave (the no-escape area).
Figure 1 shows the tracing area for the case where the hunting ground $X$ is the entire plane and the trajectory of the searching object $A$ is a straight line.

§4. Mathematical model of the search game on the cylinder

We shall consider the search problem with simple motion of the objects on the surface of an infinite circular cylinder $C$. By that we mean that the mathematical model of the game is described by the following relations:

$$a = u, \quad b = v,$$
where $a \in C$, $b \in C$, $u \in S^2_\alpha$, $v \in S^2_\beta$

(here $S^2_\alpha$ ($S^2_\beta$) is the sphere of radius $\alpha$ ($\beta$) centered at the initial point), and the object $S$ is considered detected, if at a certain instant $T$ the following inequality becomes valid:

$$\text{dist}_C(a(T), b(T)) \leq 1,$$

where $\text{dist}_C$ denotes the geodesic distance calculated along the cylinder $C$.

§5. Detection conditions on the cylinder

Let $r$ be the radius of the directing circle of cylinder $C$. In [7], [9] it was shown that successful search (detection) is possible, if either

$$l \geq \pi r$$

(1)

(the corresponding trajectory of the searching object $A$ is a ruling of the cylinder) or if the two following conditions:

$$l < \pi r,$$

$$\cos \lambda < \sin(\lambda - \mu)$$

(2) (3)

both hold, where

$$\lambda = \arccos \frac{\beta}{\alpha}, \quad \mu = \arccos \frac{l}{\pi r}$$

(the corresponding trajectory $\Gamma_0$ of $A$ is a helix that crosses the directing circles of the cylinder $C$ at the angle $\nu_0 = \lambda - \mu$).

Remark 1. Suppose the searching object $A$ moves along a helix. Then it is not difficult to verify that the tracing areas $\omega'$ and $\omega''$ that appear at arbitrarily chosen instants of time $t'$ and $t''$, respectively, can be fitted on each other by a certain (intrinsic) isometry of the cylinder $C$ onto itself.